Algorithmic Algebra and Geometry

Summer Program for Graduate Students of MSRI Sponsoring Institutions

July 6-17, 1998

Main Lectures:

- *Polytopes and Toric Varieties,* Dave Bayer, Barnard College
- *(Elementary) Algebraic Geometry from an algorithmic point of view,* Sorin Popescu, Columbia University

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Prerequisites:

- D. Cox, J. Little, D. O'Shea: *Ideals, Varieties and Algorithms*, Second Edition, Springer Verlag 1997
  We basically would like students to make a first stab at reading the entire book. However, the most important sections in order of importance are

  2. Groebner Bases. Sections 1-6.
  3. Elimination Theory. Sections 1,2 .
  4. The Algebra-Geometry Dictionary. Entire chapter.
  5. Polynomial and Rational Functions on a Variety. Sections 1,2,3,4,5.
  7. Invariant Theory of Finite Groups. Section 1.
  8. Projective Algebraic Geometry. Entire chapter.
  9. The dimension of a variety. Sections 1,2,3,4,6.
  A. Some Concepts from Algebra. Entire appendix.

References:

- G. Ewald: *Combinatorial Convexity and Algebraic Geometry*, Springer Verlag 1996
- B. Sturmfels: *Groebner Bases and Convex Polytopes*, University Lecture Notes Series Vol **8**, AMS 1995
- J. R. Munkres: *Elements of Algebraic Topology*, Addison-Wesley 1984
- D. Eisenbud: *Commutative Algebra with a view toward Algebraic Geometry*, Springer Verlag 1995
- R. Hartshorne: *Algebraic Geometry*, Springer Verlag 1977

The emergence of computational methods in algebraic geometry has lead to interactions with a number of other areas, such as combinatorics, optimization, statistics, and splines. This workshop seeks to introduce participants to these ideas and topics. *Ideals, Varieties and Algorithms*, an undergraduate text with a focus on computational methods, will be assumed as a prerequisite for the main lectures. A copy of the second edition of the book will be distributed to all pre-registered participants. This workshop will also introduce the use of specialized computer algebra systems such as *Macaulay*, *Macaulay2*, and *GAP*, which are aimed at these problem domains. Each student will have a desk with a computer and will be expected to work on a computational project.

In order to participate, students must be nominated by the Mathematics Department of an MSRI Sponsoring Institution.

There will be two main lectures each morning, as well as tutorials for the various computer algebra systems. A major feature of the program will be a series of working seminars organized by the students, with the participation of the senior lecturers.

Students are encouraged to propose their own topics for the working seminars. The following is a list of suggestions:

- Algorithmic number theory and cryptography
    - H. Cohen: *A course in computational algebraic number theory*, Graduate Texts in Mathematics, **138**, Springer-Verlag 1993
    - H.W., Lenstra, Jr.: Factoring integers with elliptic curves, *Ann. of Math* (2) **126** (1987), no. 3, 649--673
- Combinatorics - The Upper Bound Conjecture
    - R. Stanley: *Combinatorics and Commutative Algebra*, Birkhauser 1996
    - R. Stanley: The Upper Bound Conjecture and Cohen-Macaulay Rings, *Studies in Appl. Math*, **54**, 1975, 135--142
- Polytopes - classification of polytopes with few vertices, rationality.
    - G. Ziegler: *Lectures on Polytopes*, Springer Verlag 1995, lecture 6 and its references
- Enumeration of lattice points in polytopes
    - B. Sturmfels: *Groebner Bases and Convex Polytopes*, University Lecture Notes Series Vol **8**, AMS 1995
    - M. Brion: Points entiers dans les polyedres convexes, *Ann. Sci. Ecole Norm. Sup.* (4) **21** (1988), no. 4, 653--663
    - M-N. Ishida: Polyhedral Laurent series and Brion's equalities, *Internat. J. Math.* **1** (1990), no. 3, 251--265
- Splines and Groebner bases
    - L. Billera, L.L. Rose: Groebner basis methods for multivariate splines. *Mathematical methods in computer aided geometric design (Oslo, 1988)*, 93--104, Academic Press, Boston, MA, 1989
- Statistics
    - P. Diaconis, R. Graham, B. Sturmfels: Primitive partition identities. *Combinatorics, Paul Erdos is eighty*, Vol. **2**, (Keszthely, 1993), 173--192, Bolyai Soc. Math. Stud., 2, Budapest, 1996
    - P. Diaconis, D. Eisenbud, B. Sturmfels: Lattice Walks and Primary Decomposition, preprint 1997
- Integer programming
    - B. Sturmfels: *Groebner Bases and Convex Polytopes*, University Lecture Notes Series Vol **8**, AMS 1995
    - A.Schrijver: *Theory of Linear and Integer Programming*, Wiley Interscience 1986
- Topological methods in combinatorics - shellability of simplicial complexes, duality
    - A. Bjorner: Topological methods. *Handbook of combinatorics*, Vol. 1, 2, 1819--1872, Elsevier, Amsterdam, 1995

- W. Bruns, J. Herzog: *Cohen-Macaulay rings*, Cambridge Studies in Advanced Mathematics, **39**, Cambridge University Press, Cambridge, 1993.

- Combinatorial Group theory - Computational methods
  - C. Sims: Computation with finitely presented groups, *Encyclopedia of Mathematics and its Applications*, **48** Cambridge University Press, Cambridge, 1994.
- Homotopy groups of simplicial complexes
  - A. Vince: Recognizing the 3-sphere. *Graph theory and its applications: East and West (Jinan, 1986)*, 571--583, Ann. New York Acad. Sci., **576**, New York Acad. Sci., New York, 1989
- D-modules - algorithmic methods
  - A. Galligo: Some algorithmic questions on ideals of differential operators. *EUROCAL '85, Vol. 2 (Linz, 1985),* 413--421, Lecture Notes in Comput. Sci., **204**, Springer Verlag 1985

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