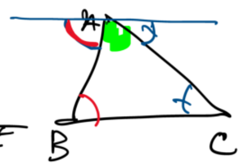
AngleSum

Wednesday, October 14, 2020 6:00 PM

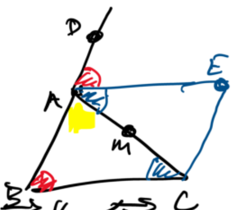
THM: IN ANY TRIANGLE, THE SUM OF THE ANGLES IS 180°

STD HIGH-SCHOOL PROOF.



DRAW A LINE PARALLEL TO $\overleftrightarrow{BC}$ THRU A, ITS A STRAIGHT ANGLE ALT. INTERIOR ANGLE TWICE, DONE

MILD VARIATION:



LET M = MIDPOINT OF $\overline{AC}$

ρ IS 180° ROTATION AROUND M, $E = \rho(B)$, $\rho(BC) = \overleftrightarrow{AE} \parallel$ TO $\overleftrightarrow{BC}$

EXTEND $\overrightarrow{BA}$ TO D.

$\overleftrightarrow{AE} \parallel \overleftrightarrow{BC}$, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CE}$

USE $\measuredangle B = \measuredangle EAD$ BY CORR. ANGLES,

$\measuredangle CAE = \measuredangle BCA$ BY ALT. INT ANGLES

$\measuredangle DAB = 180^\circ$, $\angle DAE$ ADJACENT TO $\angle CAE$

$\angle CAE$ TO $\angle CAB$.

SO $180^\circ = \measuredangle CAE + \measuredangle CAB + \measuredangle EAD$

$180^\circ = \measuredangle C + \measuredangle B + \measuredangle A$.

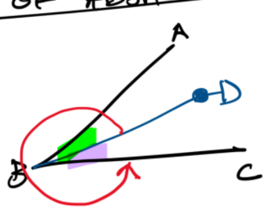
ANNOYING TECHNICAL POINT:

MUST SHOW THAT $\angle CAE$ & $\angle EAD$ ARE ADJ ANGLES, AND ALSO $\angle CAB$ AND $\angle CAE$

WRONG

BUT WHY?

TO SEE THAT TWO ANGLES ARE ADJACENT: THINK OF DEF OF ADJACENT.



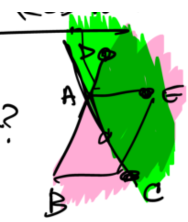
IF $\angle ABC$ AND D IS INSIDE $\angle ABC$

THEN $\angle ABD$ AND $\angle DBC$ ARE ADJACENT.

SO $\measuredangle ABC = \measuredangle ABD + \measuredangle DBC$.

ADJACENT, BUT GREEN & RED NOT.

WHY DO WE KNOW $\angle DAE$ & $\angle EAC$ ARE ADJACENT?



MUST SHOW THAT

E IS INTERIOR TO $\angle CAD$.

IE E & C ARE IN THE SAME HALFPLANE REL. TO $\overleftrightarrow{AB}$ AND

E & D IN SAME HALF REL TO $\overleftrightarrow{CA}$

E & B ARE IN OPPOSITE HALFS REL $\overleftrightarrow{AC}$ BECAUSE $\rho(\overleftrightarrow{CA}) = \overleftrightarrow{CA}$,

ROTATIONS EXCHANGE HALF PLANE.

ALSO B & D ARE IN OPP HALF PLANES

BECAUSE $B * A * D$ ON $\overleftrightarrow{AB}$ AND $\overleftrightarrow{CA} \cap \overleftrightarrow{AB} = \{A\}$.

SO E & D ARE IN SAME HALF.

ASIDE $\boxed{\frac{1}{2}(3 \cdot 2) = 3}$

$\frac{1}{2}(3 \cdot 2) = (\frac{1}{2} \cdot 3) \cdot 2$ BY ASSOC

$= (3 \cdot \frac{1}{2}) \cdot 2$ BY COMM.

$= 3 \cdot (\frac{1}{2} \cdot 2)$ BY ASSO

$= 3 \cdot 1$ BY INV

$= 3$.

$\frac{1}{2}(3 \cdot 2)$

$= \frac{1}{2} \cdot 3 \cdot 2 = 3 \cdot \frac{1}{2} \cdot 2$

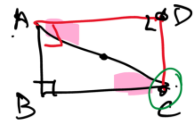
$= 3 \cdot 1$

BACK IN ORIG PROOF, STRICTLY SPEAKING, MAYBE SHOULD BE ABLE TO JUSTIFY THAT THOSE ANGLES ARE ADJACENT, SO CAN ADD.

ANOTHER PROOF!

FIRST PROVE FOR RIGHT TRIANGLES.

(IE IF $\triangle ABC$ HAS $\measuredangle B = 90^\circ$, THEN $\measuredangle A + \measuredangle B + \measuredangle C = 180^\circ$)



ROTATE $\triangle ABC$ AROUND MIDPOINT OF $\overline{AC}$ TO GET $\triangle ADC$

$\rho(B) = D$ $\square ABCD$ IS A RECTANGLE. (SINCE SIDES //)

C IS INTERIOR TO $\angle BAD$, SO $\angle BAC$ AND $\angle CAD$ ARE ADJACENT.

SO $\measuredangle DAC + \measuredangle CAB = 90^\circ$

SO $\measuredangle ACB + \measuredangle CAB = 90^\circ$ ☒

NOW FOR GENERAL $\triangle$.

3 CASES

① $B * D * C$ ② $B * C * D$ ③ $D * B * C$ SAME ISH

IN CASE ①

$\measuredangle B + \measuredangle N = 90^\circ$ FROM BEFORE

$\measuredangle C + \measuredangle M = 90^\circ$ " "

$\measuredangle B + \measuredangle C + \underbrace{\measuredangle N + \measuredangle M}_{\measuredangle A} = 180^\circ$ SO ① IS DONE.

OBSERVE

Ⓒ $\measuredangle h + \measuredangle s = 90^\circ$ FROM BEFORE

$*$ $\measuredangle B + \measuredangle BAD = 90^\circ$ FROM BEFORE

SO $\measuredangle B + \measuredangle BAD - (\measuredangle h + \measuredangle s) = 0^\circ$ $\leftarrow * -$ Ⓒ

$= \measuredangle B + (\measuredangle f + \measuredangle h) - (\measuredangle h + \measuredangle s) = 0$

$\measuredangle B + \measuredangle f = \measuredangle s$

BUT $\measuredangle BCA + \measuredangle s = 180^\circ$

SO $\measuredangle s = 180^\circ - \measuredangle BCA$

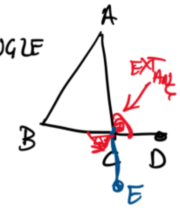
$\measuredangle B + \measuredangle f = 180^\circ - \measuredangle BCA$

$\measuredangle B + \measuredangle f + \measuredangle BCA = 180^\circ$ ☒

CASE ③ BASICALLY SAME.

BURIED HERE IS IDEA OF EXTERIOR ANGLE

DEF: IN $\triangle ABC$ WITH D ON $\overleftrightarrow{BC}$ SO THAT $B * C * D$ THEN $\angle ACD$ IS AN EXTERIOR ANGLE TO $\triangle ABC$ AT C



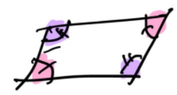
ALSO $\angle BCE$ IS EXT AT C

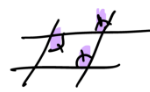
BUT $\measuredangle BCE = \measuredangle DCA$

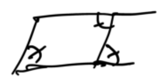
COR: (EXTERIOR ANGLE THM)

THE MEASURE OF AN EXTERIOR ANGLE TO A TRIANGLE IS THE SUM OF MEASURES OF OPPOSITE INT. ANGLES

COR: A QUADRILATERAL IS A PARALLELOGRAM $\Leftrightarrow$ ANGLES AT OPP. VERTICES ARE PAIRWISE EQUAL



$\Rightarrow$  CORR. ANGLES, ALT. INT.

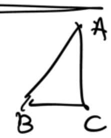
$\Leftarrow$  IF ANGLES MATCH, GET //.

DEF: IN $\triangle ABC$:

$\overline{BC}$ IS THE SIDE FACING A

$\overline{AC}$ FACES B

$\overline{AB}$ FACES C.



THM: IN ANY TRIANGLE THE ANGLE FACING THE LONGER SIDE HAS LARGER MEASURE AND CONVERSELY.

EG: $|AC| > |AB| \Longleftrightarrow \measuredangle B > \measuredangle C$

$\Longrightarrow$

Pf/ SPOSE $|AC| > |AB|$

THEN THERE IS D ON $\overline{AC}$ WITH $|AD| = |AB|$

(DONT REALLY HAVE TO SAY WHY $D \in \overline{AC}$ BUT YOU COULD, IF $\overline{AD} \cap \overleftrightarrow{BC}$ THEN $D = C$)

PICK UP HERE NEXT TIME.