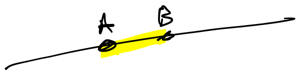
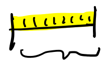
Length, Angles, measure
Wednesday, September 2, 2020 7:49 AM

(I WILL POST HOMEWORK TONIGHT)



HOW LONG IS $\overline{AB}$

BASIC IDEA IS JUST TO PICK SOME SEGMENT AND CALL IT LENGTH $= 1$.

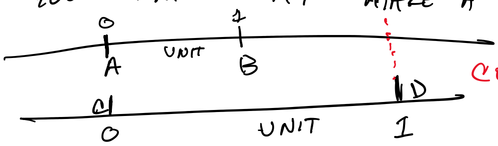


THIS IS THE UNIT SEGMENT

IF I WANT TO MEASURE SOMETHING - PICK UP RULE & COMPARE

(SOON, USE ISOMETRY ... TO MOVE THINGS AROUND)

YOU MIGHT SAY "MAKE A # LINE"



CONFLICT.

NEED OUR # LINES TO USE "SAME SCALE"

(LS) TO EVERY PAIR OF POINTS A, B IN THE PLANE, ASSIGN A NUMBER $dist(A,B)$ SATISFYING:

(i) $dist(A,B) = dist(B,A)$

(ii) GIVEN ANY RAY $\mathcal{R}$ WITH VERTEX A AND ANY $r > 0$, THEN $\exists$ EXACTLY ONE POINT B ON $\mathcal{R}$ SO THAT $dist(A,B) = r$

(iii) $dist(A,B) \geq 0$ AND $dist(A,B) > 0 \iff A \neq B$

(iv) IF A, B, C COLINEAR (IE A, B, C ALL ON A LINE) WITH $A * C * B$ THEN

$$dist(A,B) = dist(A,C) + dist(C,B)$$

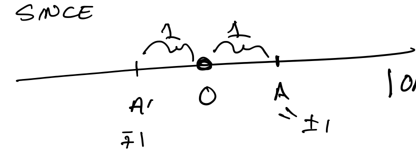
dist(,) IS A METRIC.

TYPICALLY WRITE $|AB|$ FOR $dist(A,B)$

$|AB|$ IS THE LENGTH OF $\overline{AB}$

DEF THE CIRCLE WITH CENTER O AND RADIUS $r > 0$ IS $\{P \mid dist(O,P) = r\}$

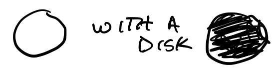
CONSEQUENCE OF (LS) IS THAT IF YOU PICK A POINT O ON A LINE, GIVES ME ONLY 2 POSSIBLE # LINES SINCE



$|OA| = |OA'| = 1$ HAS 2 SOL'NS.

COMMON BAD TERMINOLOGY IS CONFUSING

A CIRCLE WITH A DISK



IE THE CENTER OF THE CIRCLE IS NOT PART OF THE CIRCLE.

IE DOESN'T INCLUDE THE "INTERIOR."

WHAT IS THE INTERIOR?

SHIKHA ASKS: SO WHAT DOES "AREA OF A CIRCLE MEAN?"

ANSWER: NOT THAT. (STILL NEED TO DEFINE AREA)

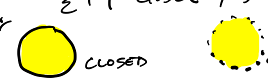
AREA OF CIRCLE IS SHORTHAND FOR "AREA OF THE INTERIOR OF THE CIRCLE"

DEF THE INTERIOR OF A CIRCLE W/ CENTER O & RADIUS r IS

$$\{P \mid dist(P,O) \leq r\}$$

ALSO CALLED THE CLOSED DISK ABOUT O WITH RADIUS r

$\{P \mid dist(P,O) < r\}$ IS OPEN DISK



CLOSED OPEN.

CIRCLE OF RADIUS 1 IS A UNIT CIRCLE

DISK ~~OF RAD 1~~ IS A UNIT DISK

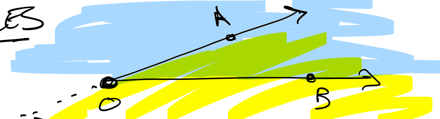
GIVEN A CIRCLE $\mathcal{C}$ OF RADIUS r AROUND O

P IS INSIDE $\mathcal{C}$ IF $dist(O,P) \leq r$

Q IS EXTERIOR TO $\mathcal{C}$ IF $dist(O,P) > r$

"Q IS IN THE EXTERIOR OF $\mathcal{C}$"

ANGLES



GIVEN $\exists$ NON-COLINEAR POINTS O, A, B

$\Rightarrow$ $\overrightarrow{OA}$, $\overrightarrow{OB}$ RAYS

LINES $\overleftrightarrow{OA}$ & $\overleftrightarrow{OB}$ DEFINE

CLOSED HALF PLANE OF $\overleftrightarrow{OA}$ CONTAINING B

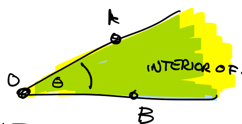
& CLOSED HALF PLANE OF $\overleftrightarrow{OB}$ CONTAINING A

INTERSECTION IS CONVEX SET & CLOSED

(CONVEX) ANGLE

$\angle AOB$ (IE ANGLES HAVE AREA...)

(SOMETIMES $\angle O$ OR θ)



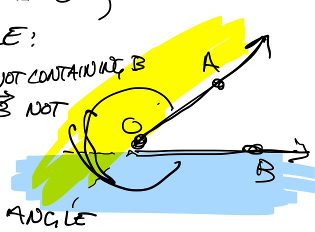
THE OTHER ANGLE:

CLOSED HALF PLANE OF $\overleftrightarrow{OA}$ NOT CONTAINING B

& CLOSED $\frac{1}{2}$PLANE OF $\overleftrightarrow{OB}$ NOT CONTAINING A

TAKE UNION.

THE (NONCONVEX) ANGLE



$\angle AOB$ ALSO. (USUALLY WE CHOOSE $\angle AOB$ TO BE THE CONVEX ONE)

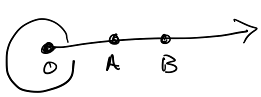
BOTH ARE THE ANGLE DETERMINED BY RAYS $\overrightarrow{OA}$ AND $\overrightarrow{OB}$

DEF

- THE RAYS ARE THE SIDES OF $\angle AOB$
- ANGLE WITHOUT SIDES IS THE INTERIOR OF $\angle AOB$
- POINT O IS THE VERTEX OF $\angle AOB$

(CAN CALL $\angle AOB$ BY $\angle O$)

WHAT IF O, A, B COLINEAR?



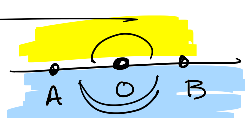
RAY $\overrightarrow{OA}$ = RAY $\overrightarrow{OB}$

IE A & B ON SAME SIDE OF O

GIVES TWO ANGLES

- FULL ANGLE, ANGLE IS PLANE
- ZERO ANGLE $\Rightarrow$ ANGLE IS RAY $\overrightarrow{OA}$

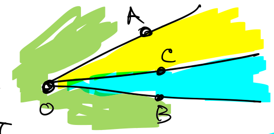
OR IF $A * O * B$



BOTH ANGLES ARE HALF PLANE

$\Rightarrow$ STRAIGHT ANGLE

ANGLES WHICH SHARE A COMMON SIDE ARE ADJACENT ANGLES.



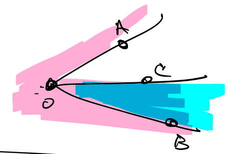
$\angle AOC$ ADJACENT TO $\angle COB$

(THE OTHER $\angle AOC$ ISN'T.)

OTHER ANGLE ADJACENT TO CONVEX $\angle COB$ IS NONCONVEX $\angle AOB$.

TRYING TO SAY THAT NONCONVEX $\angle AOC$ IS ADJACENT TO CONVEX $\angle COB$

$\Rightarrow$ TROUBLE CUZ OVERLAP.



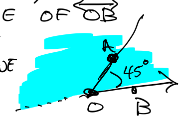
DEGREE MEASURE OF ANGLE.

(L6) TO EACH ANGLE $\angle AOB$, WE ASSOCIATE A REAL NUMBER CALLED THE DEGREE MEASURE OF $\angle AOB$ (WRITE $\measuredangle AOB$, $m\angle AOB$, $|\angle AOB|$)

(i) $0° \leq \measuredangle AOB \leq 360°$

(ii) GIVEN $\overrightarrow{OB}$ AND $0 < x < 360$, $x \neq 180$ AND A CLOSED HALF PLANE OF $\overleftrightarrow{OB}$ THEN THERE IS A UNIQUE RAY $\overrightarrow{OA}$ IN THE HALFPLANE SO THAT $\measuredangle AOB = x°$

CAN CET ALL ANGLES



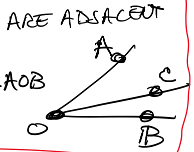
(iii) $\measuredangle AOB = 0°$ IS A ZERO ANGLE

$\measuredangle AOB = 180°$ IS A STRAIGHT ANGLE

$\measuredangle AOB = 360°$ IS A FULL ANGLE

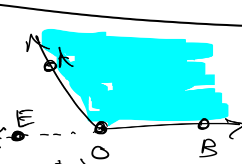
(iv) IF $\angle AOC$ AND $\angle COB$ ARE ADJACENT TO $\angle AOB$ THEN

$\measuredangle AOC + \measuredangle COB = \measuredangle AOB$



LEMMA $\angle AOB$ IS CONVEX $\iff \measuredangle AOB \leq 180^\circ$

Pf/ $\Rightarrow$ SINCE $\angle AOB$ IS CONVEX IT IS THE INTERSECTION OF TWO HALFPLANES.



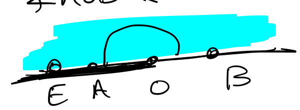
EXTEND $\overrightarrow{OB}$ TO LINE $\overleftrightarrow{OB}$ TAKE E WITH $E*O*B$

$\angle EOA$ ADJACENT TO $\angle AOB$.

BUT $\angle EOB$ IS A STRAIGHT ANGLE

$$\underbrace{\measuredangle EOA}_{\geq 0^\circ} + \measuredangle AOB = \measuredangle EOB = 180^\circ$$

So $\measuredangle AOB \leq 180^\circ$



PROOF STILL WORKS IF $\measuredangle AOB = 180^\circ$

$\Leftarrow$ SPOZE $\measuredangle AOB \leq 180$, SHOW IT IS CONVEX

IF $\measuredangle AOB = 180^\circ$, IT IS HALF PLANE SO CONVEX.

SO SPOZE $\measuredangle AOB < 180^\circ$ & NOT CONVEX & GET CONTRADICTION.

READ IN NOTES, ASK IF CONFUSED.