MAT 331 Project 2
Fun on a Swing
part 1

```
> with(share): readshare(ODE,plots):
```

We tell maple about the system of differential equations which describe the swing, and give values to all of the constants. Here we have a weight of mass m on rigid pendulum of length l .
This model accounts for both friction (given by the

$$-\frac{\varepsilon\,\nu}{m}$$

term) and for "pumping"

$$-4\,\frac{\arctan\left(\frac{f}{l}\right)\arctan(5\,\nu)}{\pi}$$

achieved by moving the mass f units from the end of the rod (the above term gives the angular offset achieved by this motion).

```
> thdot :=  (t,theta,nu) -> nu;
  nudot:=  (t,theta,nu) -> -g/l * sin(theta -2/Pi*arctan(f/l)*arctan(5*nu)) - epsilon/m * nu;

  m:=150;
  g:= 32;
  epsilon := 4;
  f := 1;

  l := 10;
```

$$thdot := (t, \theta, \nu) \rightarrow \nu$$

$$nudot := (t, \theta, \nu) \rightarrow -\frac{g\sin\left(\theta - 2\,\frac{\arctan\left(\frac{f}{l}\right)\arctan(5\,\nu)}{\pi}\right)}{l} - \frac{\varepsilon\,\nu}{m}$$

$$m := 150$$

$$g := 32$$

$$\varepsilon := 4$$

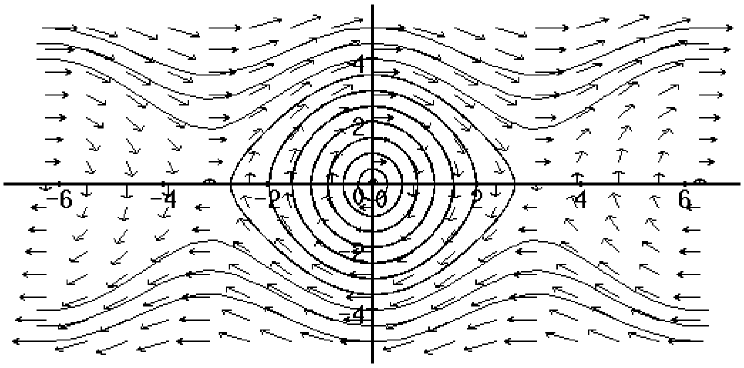
$$f := 1$$

$$l := 10$$

Now we are set to make a picture or two of the motion of the swing.

First, lets make a phase plot of the case with no friction, and no pumping. To remove those forces, we set epsilon and f to zero.

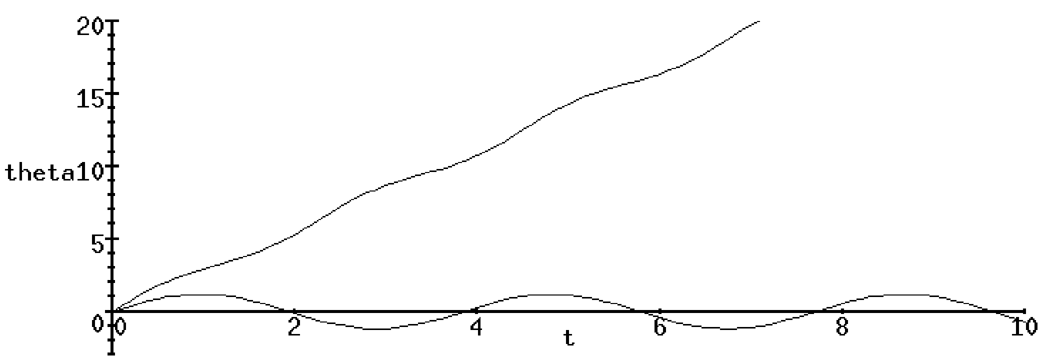
```
> epsilon:=0:
  f:=0:
  phaseplot([thdot, nudot],-2*Pi..2*Pi,-5..5,{seq([0,0,0.5*nu], nu=-10..10)},
            stepsize=0.1,numsteps=200);
```



Here we can see a number of solutions for which the swing oscillates (the elliptical solutions in the middle), 3 solutions corresponding to continuous clockwise motion (the "running solutions" at the top), and 3 with continuous counterclockwise solutions at the bottom.

Here is a plot of the angle as a function of time for two such swings starting with an initial angle of 0. One has an initial angular velocity of 4 (which is a running solution, so the angle continues to increase), and the other with an initial angular velocity of 2 (which oscillates).

```
> plot({makelist(rungekuttahf([thdot,nudot],[0,0,4],0.1,100),1,2),
       makelist(rungekuttahf([thdot,nudot],[0,0,2],0.1,100),1,2)}, t=0..10, theta=-3..20)
  ;
```



We can write a maple procedure to animate these solutions, so we can see the swing actually moving. This takes as input the result of a call to rungekuttahf, and shows a series of plots in sequence. This can be rather memory and time consuming to run, because of the large number of plots it computes.

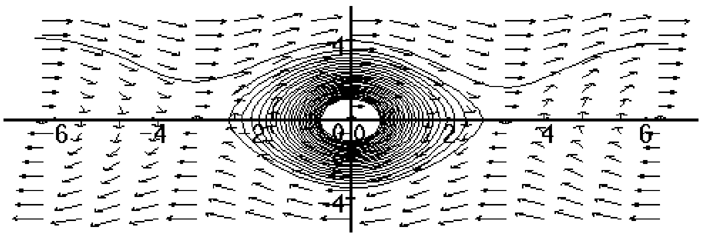
```
> SwingMovie := proc (theTrajectory)
    local numframes, i, theTitle;

    numframes := op(2,op(2,eval(theTrajectory)));
    theTitle := `swing with l=` .l;

    plots[display]([
      seq(
        plot([0,0,  l*sin(theTrajectory[i][2]), -l*cos(theTrajectory[i][2])],
              -l-1..l+1, -l-1..l+1, scaling=constrained, axes=boxed,
              thickness=2,title=theTitle),
         i=0..numframes)],
      insequence=true);

  end:
```

Here are two samples, namely, a movie for each of the two swings above:

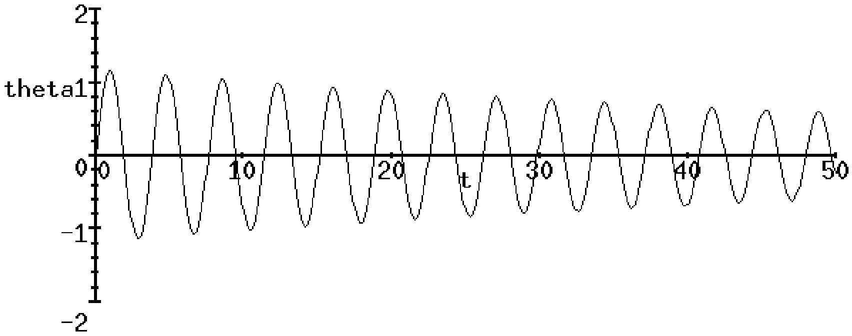
```
> SwingMovie(rungekuttahf([thdot,nudot],[0,0,4],0.1,100));
> SwingMovie(rungekuttahf([thdot,nudot],[0,0,2],0.1,100));
```

Now, lets add a little friction and see how things change.

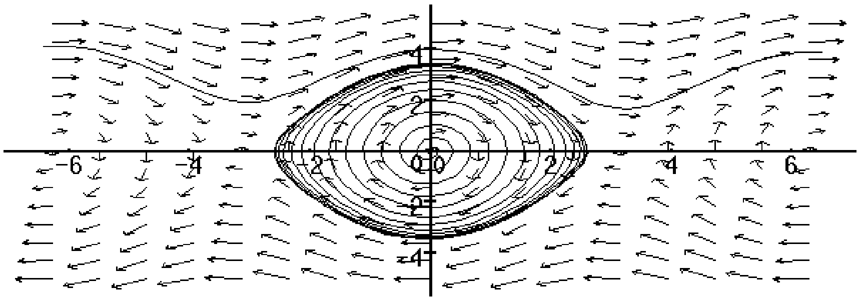
```
> epsilon:=4:
  f:=0:
  phaseplot([thdot, nudot],-2*Pi..2*Pi,-5..5,{[0,0,4],[0,0,2]},
                stepsize=0.1,numsteps=500);
```



Here, we show only the two solutions investigated above, namely [0,0,4] and [0,0,2], and we run them for a longer time (50 time steps). Note that the oscilatory solution winds down toward 0. What happens to the other one?
You could use either

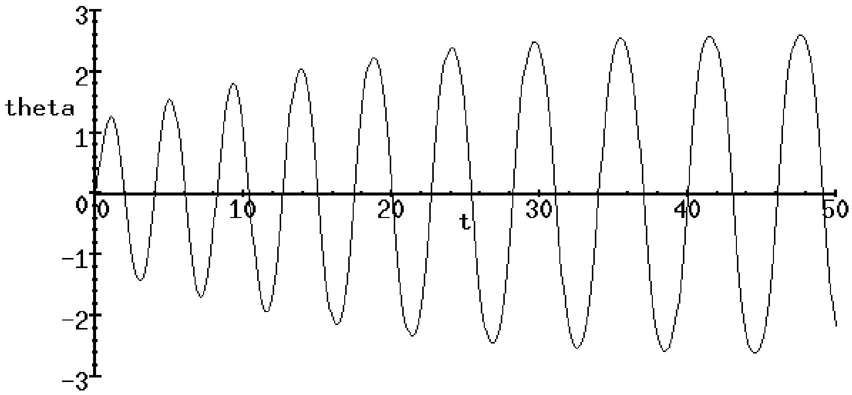
```
> plot({makelist(rungekuttahf([thdot,nudot],[0,0,4],0.1,500),1,2);
```
or
```
> SwingMovie(rungekuttahf([thdot,nudot],[0,0,4],.1,500));
```
(or both) to find out, but I'll leave that to you. Here is a graph of theta for the inner solution:
```
> nopump := rungekuttahf([thdot,nudot],[0,0,2],.1,500):
  plot(makelist(nopump,1,2),t=0..50,theta=-2..2);
```


Now, lets allow the swinger to start pumping. Once again, we take our equations, but set f to be nonzero.

```
> epsilon:=4:
  f:=1:
  phaseplot([thdot, nudot],-2*Pi..2*Pi,-5..5,{[0,0,4],[0,0,2]},
            stepsize=0.1,numsteps=500);
```


What is different here? At first glance, it looks like the solution starting at [0,0,2] goes to 0 even quicker, but a closer look shows that the spiral moves OUTWARDS. This is because phaseplot is showing us the past history, as well. Lets make a graph of the angle as a function of time, to illustrate this.

```
> pump := rungekuttahf([thdot,nudot],[0,0,2],.1,500):
  plot(makelist(pump,1,2),t=0..50,theta=-3..3);
```



You could compare the two graphs using

```
> plot(pump, nopump);
```

if you like.

```
>
```