MAT331 Exercises, Spring 12

set number 1

NOTE: *Each exercise is worth 10 points and can be turned in at any time before its "expiration date". At the end of the semester, I will expect you to have turned in at least 1/2 of the exercises assigned. If you do more, I will pick your best grades. If you do less, the missing grades will be counted as zeros. Altogether, these will count the same as one project.*

Many of these problems will require you to use the help system and/or read the text to figure out what commands you need to use and how to use them.

1. (*expires 2/2*) If $p(x)$ is a complex polynomial with real coefficients, it is well known that it can be factored into a product of linear and quadratic terms with real coefficients, or into a product of linear terms only if the coefficients are allowed to be complex.

   Use `Maple` to write $p(x) = x^5 - 2x^4 - 10x^3 + 20x^2 - 16x + 32$ as a product of *exact* linear and quadratic factors with real coefficients. By exact, I mean you should leave any non-rational factors expressed as radicals; do not approximate terms like $\sqrt{3}$ as 1.73205, etc.

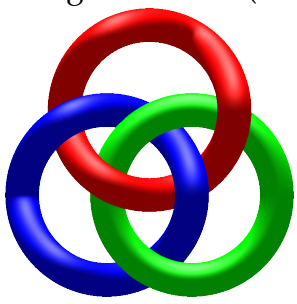
   Also write $p(x)$ as a product of only linear factors.

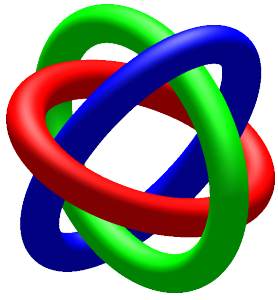
2. (*expires 2/2*) Draw a graph showing both $\cos(x)$ and its fifth Taylor polynomial (that is, $1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4$) for x between -4 and 4. What degree of Taylor polynomial seems to be needed to get good agreement in this range? *Hint: use a variation of the command* `convert(taylor(cos(x),x,5),polynom)` *to make this work. Think of a suitable way to demonstrate that the approximation you have taken is "good"– what is a good definition of "good" here? Be sure to* **explain** *your choice of the meaning of "good".*

3. (*expires 2/9*) The Borromean rings are a set of three mutually interlocked rings, arranged so that if one ring is removed, the other two become unlinked.

   Use `Maple` to make an image of the Borromean rings in $\mathbb{R}^3$. *Hint: I suggest using* `tubeplot` *from the* `plots` *package. The real challenge of this problem is figuring out how to parameterize the three curves.*

   Note that the Borromean rings can not be made from flat, round rings. Either some of the rings have to be bent a little to weave through each other, or flat rings which are elliptical can be used. See the figures below (made with `Maple`).





4. (*expires 2/9*) Consider the planar curve γ defined by $x^2y^3 + y^2 + y - 2e^x = 0$. Using **only** `Maple`, find the slope of the tangent line to the curve at $(0, 1)$. Then plot the curve and the tangent line on the same graph.
   Hint: you might want to use `implicitplot` *from the library* `plots`. *You might find* `implicitdiff` *helpful, too.*

5. (*expires 2/9*) Plot the function $f(x) = 2\sin x - x^3 - 1/5$, for $x \in [-4, 4]$. Find all the zeros of the function with an accuracy of 20 decimal digits. *Hint: See* `Digits`, `fsolve`.