MAT331 homework problems

13. (*expires 6 March*) Find all the solutions to the differential equation

$$\frac{dx}{dt}(t) = -2x(t), \quad t \in \mathbb{R}.$$

Among them, single out the one for which $x(0) = 3$. [*Hint: read the help page for* `dsolve`, *or just do it in your head. It is that easy.*]

14. (*expires 6 March*) Have `Maple` find analytic solutions to the following system of differential equations,

$$\begin{cases} y''(t) - z(t) &= e^t, \\ z'(t) - y(t) &= 0, \end{cases}$$

with initial conditions: $y(0) = 1,\ y'(0) = 0,\ z(0) = k$. Let us denote the solutions by $y_k(t),\ z_k(t)$ (since they depend on the parameter k).

For k taking all integer values from -10 to 10, and $t \in [-4, 2]$, plot the functions y_k in blue, and the functions z_k in red, all on the same graph. (Yes, you will then have 42 functions plotted on the same graph.) [*This is certainly a case when you don't want to retype the functions that* `Maple` *finds. You will almost certainly need to read the help page for* `dsolve`. *I also found* `subs`, `unapply`, *and* `seq` *useful.*]

15. (*expires 6 March*) For the functions $y_k(t)$ and $z_k(t)$ found in problem #14, plot the parametric curves $\varphi_k(t) = [y_k(t), z_k(t)]$ for integer values of k between -5 and 5 and $-6 < t < 4$ on the same graph. Use the `view` option of plot to only show what lies in the region $-10 < y < 10, -10 < z < 10$, and use a sequence of colors so that each solution is a different color. [*HINT: you might find something like* `seq(COLOR(HUE,i/11),i=0..10)` *useful for the latter.*]

16. (*expires 6 March*) Find all the fixed points of the system

$$\begin{cases} \dot{x} &= x^2 + y, \\ \dot{y} &= x(y^2 - 1), \end{cases}$$

a fixed point being a solution for which *both* $x(t)$ *and* $y(t)$ stay constant. For each of these points, describe the behavior of the solutions that have initial conditions nearby. You can use `Maple` to figure out what happens for nearby points, or you can use more mathematical methods.

NOTE: *The fact that there are various notations for differential equations is purely intentional.*