MAT 331, Fall 2002

Project 2: Gliders and Differential Equations

Due Mon, November 11

In class, we have been discussing a system of differential equations which approximately models the flight of a glider. In this project, you are to explore a similar system, which models such a glider with a propeller or small engine attached. In this case, our model is

$$\frac{dv}{dt} = -\sin\theta - 0.3\,v^2 + k, \qquad \frac{d\theta}{dt} = \frac{v^2 - \cos\theta}{v}.$$

Here, as in class, $v > 0$ is the speed of the glider through the air (*not* the horizontal speed), and θ is the angle the nose makes with the horizontal direction. Note that we have fixed the drag coefficient to be 0.3, but have added an additional term k to account for acceleration caused by the propeller.

You are to analyze and classify the solutions of this system for all $k \geq 0$ (note that $k = 0$ was covered in class). This means that you should find the various ranges of k where the behavior is qualitatively different.

Such analysis should include a discussion of the existence of fixed points (equilibrium solutions) and their linearizations (i.e., a discussion of eigenvalues and eigenvectors and how this relates to the solutions), as well as a discussion of the long-term behavior of the solutions.

In addition to the description of the trajectories in the (θ, v)-plane, you should also relate the solutions to properties of glider flight. For example, discuss whether the glider eventually must crash or can stay aloft indefinitely, whether the glider does loops, etc.

In your writeup, you should include a number of relevant pictures and graphs, preferably (but not necessarily) produced by Maple. These pictures should be used to illustrate your exposition, not merely included without comment or reason (remember: "a picture is worth a thousand words, but a thousand pictures are worthless"). Please pay attention to clarity of exposition; **do not** merely hand in an annotated Maple worksheet. While including relevant Maple commands is useful, your goal is to explain what you are doing from a *mathematical* point of view, not to describe how to use Maple to perform a certain task. You need not (although you can) state or prove the relevant theorems, but you should explain how you are using them to do your analysis.