Math 331, Fall 2002: Problems 17-20

17. (*expires 10/28*) Consider the differential equation $\dot{\mathbf{z}}(t) = \mathbf{F}(\mathbf{z}(t))$, where the vector $\mathbf{z}(t) = (x(t), y(t))$ and the field $\mathbf{F}(x, y) = (-y, x - y)$. Plot a few solutions. What happens to them when $t \to +\infty$? Give a "`Maple`-proof" that this is a general fact for *every* solution. [*A "`Maple`-proof" is an argument that is rigorous once we accept `Maple` results as incontrovertibly true.*]

18. (*expires 10/28*) (No `Maple`.) For the equation $\dot{\mathbf{z}} = \mathbf{F}(\mathbf{z})$, $\mathbf{z} = (x, y)$, with the vector field

$$\mathbf{F}(x, y) = \left\langle -x(x^4 + y^4) - y \, , \, x - y(x^4 + y^4) \right\rangle ,$$

prove that the origin is an attractor in the future, i.e., every solution verifies

$$\lim_{t \to +\infty} \mathbf{z}(t) = 0.$$

[*You can ask around how to do this, but then you have to show clearly that you have understood it.*]

19. (*expires 10/28*) For the system of differential equations of prob. #23,

$$\begin{cases} \dot{x} &= x^2 + y, \\ \dot{y} &= x(y^2 - 1), \end{cases}$$

find the eigenvalues and eigenvectors of the Jacobian at the fixed points. [*This is a give-away if you have done #16.*]

20. (*expires 10/28*) Consider the equations of the glider with no drag term ($R = 0$). Use `dsolve, type=numeric` to solve them numerically with initial conditions $\theta(0) = 0$, $v(0) = 0.8$. Then solve exactly the linearized system around the fixed point $(\theta_0, v_0) = (0, 1)$, with the same initial conditions. Graph the two functions for $0 \le t \le 5$, and give a good estimate of their maximum difference. What happens if we take a larger t-range?