Math 331, Fall 2002: Problems 7-10

NOTE: *Each exercise is worth 10 points and can be turned in at any time before its "expiration date". At the end of the semester, I will expect you to have turned in at least 2/5 of the exercises assigned. If you do more, I will pick your best grades. If you do less, the missing grades will be counted as zeros. Altogether, these will count the same as one project.*

7. (*expires 9/30*) Fit the points $(-1.9, -4.7), (-0.8, 1.2), (0.1, 2.8), (1.4, -1.2), (1.8, -3.5)$ by means of a quadratic function $f(x) = ax^2 + bx + c$, using the least square method. First, do this step by step, as we did in class; then, use the built-in `Maple` command, described in the notes. Check that the two solutions agree.

8. (*expires 9/30*) Fit the set of points

$$(1.02, -4.30), (1.00, -2.12), (0.99, 0.52), (1.03, 2.51), (1.00, 3.34), (1.02, 5.30)$$

with a line, using the least square method we used in class. You will see that this is not a good fit. Think of a better way to do the fit and use `Maple` to do it. Explain in your solution why you think your better way is better.

9. (*expires 10/7*) *[In this problem use `Maple` only as a word processor. If you're more confortable with paper, you can turn in a paper instead of a `Maple` worksheet.]* Let n points of the form (r_i, r_i^2), $i = 1, 2, \ldots, n$, be given. What is the quadratic function $f(x) = ax^2 + bx + c$ that best fits them? **Prove** your answer. Does it depend on the optimization method (least square or others)?

10. (*expires 10/7*) Once we have calculated the line (or any other curve, for that matter) that best fits a sets of points, we can get an idea how good the fit is by plotting the line together with the points. It is much more scientific, however, to have a measure for this. Come up with a function of the data and parameters of a given best-fit problem that is small when the fit is good and large when the fit is bad, no matter how many points are used. Justify your answer.