Midterm 2

MAT 127

Oct 29, 2018

Name: (please print)	**ID #**:
Your section:	(see list below)

	1 20pt	2 10pt	3 10pt	4 10pt	5 10pt	**Total** 60pts
Grade						

No notes, books or calculators.

You must show your reasoning, not just the answer. Answers without justification will get only partial credit.

Please cross out anything that is not part of your solution — e.g., some preliminary computations that you didn't need.

Lecture 1	MWF 10:00am-10:53am	Chuanhao Wei
Lecture 2	MF 1:00 pm – 2:20 pm	Jingrui Cheng
Lecture 3	TuTh 10:00 am – 11:20 am	Sabyasachi Mukherjee
Lecture 4	TuTh 5:30 pm – 6:50 pm	Babak Modami

1. *(20 pts)*

(a) Calculate the degree 3 Taylor polynomial $T_3(x)$ of $f(x) = \sqrt{x}$ centered at $a = 1$.

$f(x) = \sqrt{x} = x^{1/2}$ $\quad f(1) = 1$ $\quad$ TAYLOR COEFFS: 1

$f'(x) = \frac{1}{2}x^{-1/2}$ $\quad f'(1) = 1/2$ $\quad 1/2$

$f''(x) = -\frac{1}{4}x^{-3/2}$ $\quad f''(1) = -\frac{1}{4}$ $\quad \frac{-1}{4 \cdot 2!} = -1/8$

$f'''(x) = \frac{3}{8}x^{-5/2}$ $\quad f'''(1) = \frac{3}{8}$ $\quad \frac{3}{8 \cdot 3!} = \frac{3}{48} = \frac{1}{16}$

SO

$$T_3(x) = 1 + \frac{(x-1)}{2} - \frac{(x-1)^2}{8} + \frac{(x-1)^3}{16}$$

(b) Approximate $\sqrt{2}$ by the Taylor polynomial computed in the previous part. (No need to compute error bound.)

$$T_3(2) = 1 + \frac{1}{2} - \frac{1}{8} + \frac{1}{8} = \frac{3}{2}$$

NOT GREAT: $T_3(2) = 1.5$

$\sqrt{2} \approx 1.414$

2. *(10 pts)*

Using the Maclaurin series expansions of e^x and $\cos(x)$ given below

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots,$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots,$$

find the first three non-zero terms of the Maclaurin series of $f(x) = e^x \cos(2x)$.

$$\left(1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots\right)\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots\right)$$

$$= 1 + x + \left(\frac{x^2}{2} - \frac{x^2}{2!}\right) + \left(\frac{x^3}{3!} - \frac{x^3}{3!}\right)$$

$$+ \left(\frac{x^4}{4!} - \frac{x^4}{4} + \frac{x^4}{4!}\right) + \text{TERMS IN } x^5 \text{ AND HIGHER}$$

$$= 1 + x + 0 + 0 + \left(\frac{2x^4}{4 \cdot 3 \cdot 2} - \frac{3x^4}{3 \cdot 4}\right) + \cdots$$

$$= \boxed{1 + x - \frac{x^4}{6}} + \mathcal{O}(x^5)$$