MAT 127 Midterm

Morning of March 25, 2021

Name: _____ **ID:** _____

There are 5 pages (plus this cover sheet) in this exam; each contains a question (and the first "question" is at the bottom of this page). Make sure that you have them all.

Your solution to each problem should include all intermediate steps and be presented in an orderly fashion; be sure to cross out any work that the grader should ignore. **Books, calculators, use of the internet, and discussions with friends are not permitted.** You may use a single sheet of (handwritten) notes, written on the front and the back.

You are welcome to read Prof. Sutherland's mind in order to obtain the solutions to this exam. Remember, however, that his algebra and arithmetic skills are terrible, and he's probably made at least one mistake on each problem. (If you have done this, please make an appropriate acknowledgment to avoid plagiarism issues.)

Points will be taken off for writing mathematically false statements, even if the rest of the problem is correct.

Leave all answers in exact form (that is, do *not* approximate π, square roots, and so on.)

You have **80 minutes** to complete this exam.

1. Copy the following statement and **sign your name below it**.

I attest that all the work done on this exam is my own, and I did not consult any other people or resources (internet or otherwise) except those explicitly allowed while doing these problems.

Signed: _____

10 pts 2. Consider the sequence $\{a_n\}$ which begins

$$\frac{\sin(3)}{3}, \frac{\sin(5)}{9}, \frac{\sin(7)}{27}, \frac{\sin(9)}{81}, \frac{\sin(11)}{243}, \ldots$$

(a) Assuming the pattern continues, give an explicit formula a_n in the sequence above. Be sure to state the initial value of n.

(b) Determine whether the sequence converges or diverges, fully justifying your answer. If it converges, give the limit it converges to.

10 pts 3. Find the radius and interval of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{3^n \sqrt{n}} (x-3)^n \quad .$$

Be sure to check the endpoints of the interval.

10 pts

4. The fair princess Aracelushka is being held prisoner by the evil monkey-king. As a signal to her brother Jack, she drops an enchanted orb out of the window of the tower where she is being held, 120 feet above the ground. Each time the orb strikes the ground, it sends out a beacon of golden light, then bounces and returns to a height two-thirds of its previous maximum height. What is the total vertical distance traveled by the orb if it bounces infinitely many times?

5. Let $f(x) = \dfrac{2}{1+4x^3}$.

5 pts (a) Write the Maclaurin series (ie, the Taylor series about 0) for $f(x)$.

5 pts (b) Write the Maclaurin series for $\int f(x)\,dx$.

5 pts (c) Would it be reasonable to use the first few nonzero terms from your answer to the previous part to estimate $\int_0^1 f(x)\,dx$. Why or why not? Explain. (You don't have to estimate this value, just explain why or why not it would make sense to try.)

15 pts

6. For each of the infinite series below, determine whether they converge or diverge. State explicitly how you determined this, and justify your answers fully.

(a) $\sum_{n=0}^{\infty} n\pi$

(b) $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2}$

(c) $\sum_{n=0}^{\infty} \frac{n^3+n}{n^5-n^3+2}$