EXERCISE ONE Determine whether the following series are convergent:

$$\sum_{n=1}^{\infty} \frac{n^2}{\sqrt{n^8 - n + 1}}$$

and

$$\sum_{n=1}^{\infty} \frac{1}{e^n + n^{\frac{3}{5}}}.$$

$$\lim \frac{\frac{n^2}{\sqrt{n^8-n+1}}}{\frac{1}{n^2}} = \lim \frac{n^4}{\sqrt{n^8 - n + 1}} = \lim \frac{1}{\sqrt{1 - n^{-7} + n^{-8}}} = 1$$

Since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges, the first series converges by the Limit Comparison Test.

We have

$$\frac{1}{e^n + n^{\frac{3}{5}}} < \frac{1}{e^n}.$$

The series $\sum_{n=1}^{\infty} \frac{1}{e^n}$ converges because it is a geometric series with ration of absolute value < 1. Thus the second series converges by the Comparison Test.