EXERCISE ONE Consider the following differential equation:

$$\frac{y'' + 6y'}{18} = -y.$$

1. Find the general solution of this differential equation. (5 points)
2. Find the particular solution y with initial conditions $y(0) = 5, y'(0) = 3$. (4 points) What is the limit of $y(x)$ when x goes to $+\infty$? (1 point)

The characteristic equation $z^2 + 6z + 18z = 0$ has two complex roots $-3 \pm 3i$. Thus the general solution of the differential equation is $C_1 e^{-3x} \cos(3x) + C_2 e^{-3x} \sin(3x)$. The derivative function y' is
$-3C_1 e^{-3x} \cos(3x) - 3C_1 e^{-3x} \sin(3x) - 3C_2 e^{-3x} \sin(3x) + 3C_2 e^{-3x} \cos(3x)$. The initial conditions can be written as:

$$C_1 = 5, -3C_1 + 3C_2 = 3.$$

Thus $C_1 = 5$, $C_2 = 6$ and the desired particular solution is $5e^{-3x} \cos(3x) + 6e^{-3x} \sin(3x)$.