PRINT your name:

1. Find the general solution to $y' = \dfrac{3x^2 - x}{y}$.

$\dfrac{dy}{dx} = (3x^2 - x)\left(\dfrac{1}{y}\right)$

SO SEPARATING VARIABLES GIVES

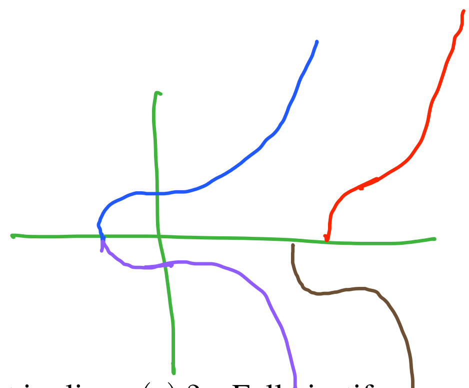
$$\int y\,dy = \int (3x^2 - x)\,dx$$

$$\frac{y^2}{2} = x^3 - \frac{x}{2} + C_0$$

$$y^2 = 2x^3 - x + C$$

$$\boxed{y = \pm\sqrt{2x^3 - x + C}}$$

NOTE THAT THERE ARE TWO BRANCHES OF SOLUTIONS, THOSE WITH $y > 0$ AND THOSE WITH $y < 0$. SOLUTIONS LOOK LIKE →



2. For the solution $y(x)$ to part 1 satisfying $y(1) = -2$, what is $\lim_{x\to+\infty} y(x)$? Fully justify your answer.

If the limit is not a real number, distinguish between $+\infty$, $-\infty$, and other behavior.

FIRST, LETS DO THIS BY FINDING THE SOLUTION WITH $y(1) = -2$. NOTE THAT THIS MEANS WE TAKE THE NEGATIVE SQUARE ROOT, IE $y = -\sqrt{2x^3 - x + C}$.

LET $x = 1$ AND $y = -2$ TO GET $-2 = -\sqrt{2 - 1 + C}$

SO $2 - 1 + C = 4$, IE $\boxed{C = 3}$

$y = -\sqrt{2x^3 - x + 3}$.

$$\boxed{\lim_{x\to\infty} -\sqrt{2x^3 - x + 3} = -\infty}$$

SUPPOSE WE DIDN'T FIND THE GENERAL SOLUTION. HOW COULD WE KNOW $\lim_{x\to\infty} y(x) = -\infty$?

OBSERVE THAT FOR $y' = \dfrac{3x^2 - x}{y}$, IF $x > 1/3$, $3x^2 - x > 0$. SINCE $y(1) < 0$, $y' < 0$ FOR ALL $x > 1/3$, SO SOLUTIONS DECREASE AND SLOPES ARE ALWAYS LESS THAN -2.

SO $\lim_{x\to\infty} y(x) \to -\infty$. [OR, SKETCH THE SLOPE FIELD.]