PRINT your name:

Answer each question below completely. You must fully justify your answers to get credit. Even a correct answer with no justification is wrong.

1. Write a (geometric) power series which agrees with the function $\frac{1}{8+x^3}$ on the interval $-2 < x < 2$.

$$\frac{1}{8+x^3} = \frac{1/8}{1+\frac{x^3}{8}} = \frac{1/8}{1-\left(\frac{-x^3}{8}\right)} = \frac{1}{8}\sum_{n=0}^{\infty}\left(\frac{-x^3}{8}\right)^n$$

$$= \frac{1}{8}\sum_{n=0}^{\infty}\frac{(-1)^n x^{3n}}{8^n}$$

$$= \boxed{\sum_{n=0}^{\infty}\frac{(-1)^n x^{3n}}{8^{n+1}}}$$

$$= \frac{1}{8} - \frac{x^3}{8^2} + \frac{x^6}{8^3} - \frac{x^9}{8^4} + \cdots$$

2. Write a power series representation of $\frac{3x^2}{(8+x^3)^2}$.

Hint: the answer to the first question is relevant.

OBSERVE THAT $\frac{d}{dx}\left(\frac{1}{8+x^3}\right) = \frac{-3x^2}{(8+x^3)^2}$

$$\text{SO } -\frac{d}{dx}\sum_{n=0}^{\infty}\frac{(-1)^n x^{3n}}{8^{n+1}} = -\left(-\frac{3x^2}{8^2} + \frac{6x^5}{8^3} - \frac{9x^8}{8^4} + \cdots\right)$$

$$= \frac{3x^2}{8^2} - \frac{6x^5}{8^3} + \frac{9x^8}{8^4} - \frac{12x^{11}}{8^5} + \cdots$$

$$\boxed{\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(3n)x^{3n-1}}{8^{n+1}}}$$

SAME.