MAT 127 **FINAL** **SPRING 2019**

LECTURE #:

NAME and ID #:

Problem	Points Possible	Points
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
Total	450	

INSTRUCTIONS:

1. **SHOW YOUR WORK.** Correct answers without sufficient work will receive minimal or no credit. If you use a theorem from the book, you need to tell us which one you are using: give its name (example: "Test for divergence"), or its statement (example: "if $\lim a_n \neq 0$, then $\sum a_n$ diverges").
2. Provide clearly written answers in the space provided. You can use the flip sides of the pages and page 9 as scrap paper. **Do not tear off any page!!! Loose pages will prompt us to check to verify there is no cheating. You must return all pages stapled together.**
3. No: books, notes, calculators, devices of any kind.
4. **CELL PHONES OFF AND STORED AWAY.**

1. (pts)

(a) Determine whether the series converges or diverges (justify your answer briefly):

$$\sum_{n=1} \frac{1}{n^3}.$$

(b) Determine whether the series converges or diverges (justify your answer briefly):

$$\sum_{n=1} \left(\frac{5}{4}\right)^{2n}.$$

(c) Solve the initial value problem:

$$y' = 1, \qquad y(2) = 3.$$

(d) Write a differential equation admitting $y(x) = e^x + 1$ as a solution.

2. (pts) Find the sum of the series:

$$\sum_{n=1}^{\infty} \frac{(-9)^{n-1}}{10^n}.$$

3. (pts) Determine whether the series is convergent or divergent:

$$\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2+1}.$$

4. (pts) Determine whether the series is convergent or divergent:

$$\sum_{n=1}^{\infty}(-1)^n \frac{n}{n^2+1}$$

5. (pts) Find the radius of convergence and interval of convergence of the series:

$$\sum_{n=1}^{\infty} \frac{n}{4^n}(x+1)^n.$$

6. (pts) Solve the initial value problem:

$$\frac{dy}{dx} = \frac{x}{y}, \qquad y(0) = -3.$$

7. (pts) Use Euler's method with step size 0.2 to estimate $y(0.4)$, where $y(x)$ is the solution of the initial-value problem:

$$y' = y + xy, \qquad y(0) = 1.$$

8. (pts) How many years will it take an investment to triple in value if the interest rate is 5% per year compounded continuously?

(Since you do not have a calculator, do not express the answer as a decimal number.)

9. (pts) A bacteria culture initially contains 100 cells and grows at a rate proportional to its size. After an hour the population has increased to 400. Find an expression for the number of bacteria after t hours.

10. (pts) A tank contains 1000 L (liters) of pure water. Brine that contains 0.05 kg of salt per liter of water enters the tank at a rate of 5 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate of 5 L/min. How much salt is in the tank after t minutes?

SCRAP PAPER

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