Quiz 7 wed.

Thursday, April 15, 2021 4:55 PM

1) VERIFY THAT $y(x) = \sin(2x-1) + (x-1)^2$ IS A SOLUTION TO

$$y'' + 4y = 4x^2 - 8x + 6.$$

WE HAVE $y' = 2\cos(2x-1) + 2(x-1)$

SO $y'' = -4\sin(2x-1) + 2$

THEN

$$y'' + 4y = (-4\sin(2x-1) + 2) + 4(\sin(2x-1) + x^2 - 2x + 1)$$

$$= 4x^2 - 8x + 4 + 2 = 4x^2 - 8x + 6$$

AS CLAIMED.

2) SOLVE THE INITIAL VALUE PROBLEM

$$y' = xe^{x^2-1} + \frac{1}{\sqrt{x}} - 3, \quad y(1) = \tfrac{1}{2}.$$

INTEGRATING BOTH SIDES GIVES

$$\int dy = \int (xe^{x^2-1} + x^{-1/2} - 3)\,dx$$

LET $u = x^2 - 1$

$du = 2x\,dx$

$$y = \frac{1}{2}e^{x^2-1} + 2x^{1/2} - 3x + C.$$

SINCE $y(1) = \frac{1}{2}$, WE HAVE

$$\cancel{\frac{1}{2}} = \cancel{\frac{1}{2}}e^0 + 2 - 3 + C$$

THAT IS, $0 = -1 + C$, SO $C = 1$.

THE DESIRED SOLUTION IS:

$$\boxed{y(x) = \frac{1}{2}e^{x^2-1} + 2\sqrt{x} - 3x + 1}$$