DETERMINE WHETHER THE SERIES BELOW CONVERGE OR DIVERGE

(a) $\sum_{n=1}^{\infty} \frac{3n^2}{5n^5+n^3+2n+1}$ CONVERGES. USE DIRECT COMPARISON TO THE p-SERIES $\sum \frac{1}{n^3}$ (WHICH CONVERGES). FOR ALL $n \geq 1$

$$\frac{3n^2}{5n^5+n^3+2n+1} < \frac{1}{n^3} \quad (\text{SINCE } 3n^5 < 5n^5+n^3+2n+1)$$

SO THE SERIES CONVERGES.

OR USE LIMIT COMPARISON:

$$\lim_{n\to\infty} \left(\frac{3n^2}{5n^5+n^3+2n+1} \cdot \frac{1}{1/n^3} \right) = \lim_{n\to\infty} \left(\frac{3n^5}{5n^5+n^3+2n+1} \right) = \frac{3}{5}.$$

SINCE $3/5 \neq 0$, THE SERIES BOTH CONVERGE.

(b) $\sum_{n=1}^{\infty} (-1)^n \frac{2}{3^n+n} = -\frac{2}{4} + \frac{2}{11} - \frac{2}{30} + \cdots$

THIS IS AN ALTERNATING SERIES.

THE TERMS DECREASE IN ABS VALUE $\left(\frac{2}{3^n+n} > \frac{2}{3^{n+1}+n} \right)$

AND $\lim_{n\to\infty} \frac{2}{3^n+n} = 0$ (SINCE $3^n \to \infty$ AS $n \to \infty$)

SO IT CONVERGES.

ALTERNATIVELY, THE SERIES CONVERGES ABSOLUTELY, SINCE $\left| (-1)^n \frac{2}{3^n+n} \right| = \frac{2}{3^n+n}$

AND $\sum_{1} \frac{2}{3^n+n}$ CONVERGES BY COMPARISON WITH $2 \sum_{1} \left(\frac{1}{3}\right)^n$