- FIND THE TAYLOR SERIES CENTERED AT 1 FOR e^{x-1}.

WE COULD TAKE DERIVATIVES AND DETERMINE THE COEFFICIENTS. BUT EASIER IS JUST TO SUBSTITUTE INTO $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

TO GET $e^{x-1} = \sum_{n=0}^{\infty} \frac{(x-1)^n}{n!} = 1 + (x-1) + \frac{(x-1)^2}{2!} + \frac{(x-1)^3}{3!} + \cdots$

- USE YOUR ANSWER TO CALCULATE

$$\lim_{x \to 1} \frac{e^{x-1} - x}{(x-1)^2}$$

REPLACING e^{x-1} BY THE SERIES ABOVE, WE GET

$$\lim_{x \to 1} \frac{\left(\cancel{1} + (\cancel{x} - \cancel{1}) + \frac{(x-1)^2}{2!} + \frac{(x-1)^3}{3!} + \cdots\right) - \cancel{x}}{(x-1)^2}$$

$$= \lim_{x \to 1} \frac{\cancel{\frac{(x-1)^2}{2}} + \frac{(x-1)^{\cancel{3}\,1}}{3!} + \cdots}{\cancel{(x-1)^2}} = \lim_{x \to 1} \left(\frac{1}{2} + \frac{(x-1)}{3!} + \cdots \right)$$

$$= \frac{1}{2}.$$

NOTE THAT DOING THIS BY L'HOPITALS REQUIRES TAKING DERIVATIVES TWICE, UNTIL WE GET A CONSTANT IN THE DENOMINATOR.