FIND THE GEOMETRIC SERIES REPRESENTATION OF

$$f(x) = \frac{3}{2+x^2}.$$

SINCE $\frac{1}{1-\cancel{x}} = \sum_{n=0}^{\infty} \cancel{x}^n$, WRITE $\frac{3}{2+x^2} = \frac{3/2}{1-(-x^2/2)} = \frac{3}{2}\frac{1}{1-(-x^2/2)}$.

NOW SUBSTITUTE $x \to -x^2/2$ TO GET

$$f(x) = \frac{3}{2+x^2} = \frac{3}{2}\sum_{n=0}^{\infty}\left(\frac{-x^2}{2}\right)^n = \frac{3}{2}\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{2^n} = \sum_{n=0}^{\infty}\frac{(-1)^n 3x^{2n}}{2^{n+1}}$$

$$= \frac{3}{2} - \frac{3x^2}{4} + \frac{3x^4}{8} - \frac{3x^6}{16} + \cdots$$

USE THE ABOVE RESULT TO DERIVE A SERIES FOR $g(x) = \frac{-6x}{(2+x^2)^2}$

SINCE $g(x) = f'(x)$, WE CAN DIFFERENTIATE THE ABOVE TO GET A SERIES FOR $g(x)$.

$$g(x) = \frac{-6x}{(2+x^2)^2} = \sum_{n=1}^{\infty}\frac{(-1)^n 6n x^{2n-1}}{2^{n+1}} = \sum_{n=1}^{\infty}\frac{(-1)^n 3n x^{2n-1}}{2^n}$$

$$= -\frac{3x}{2} + \frac{6x^3}{4} - \frac{9x^5}{8} + \frac{12x^7}{16} - \cdots$$