Stony Brook

STATE UNIVERSITY OF NEW YORK

MAT 125 Midterm 1 – Spring 2020 - PROF. WERTZ & ANDERSEN

Question	Points
1	/15
2	/10
3	/10
4	/20
5	/15
6	/10
7	/10
8	/10

Last Name: _____ First Name: _____

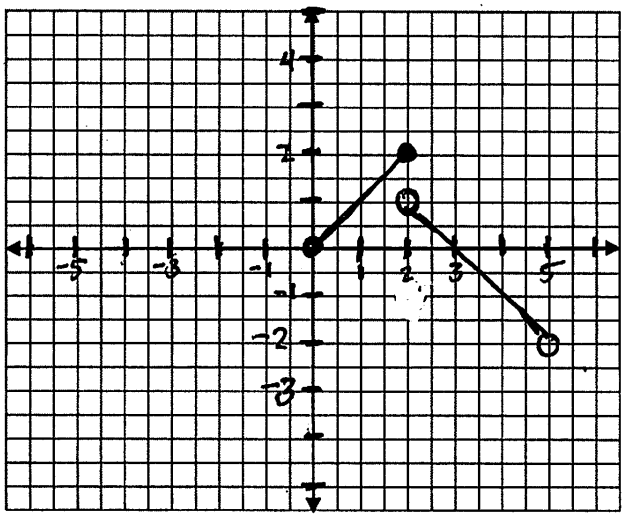
Recitation #: R_____ (see below)

R01	M	5:30pm- 6:23pm	Earth&Space	181	El Mehdi Ainasse
R02	Th	4:00pm- 4:53pm	Earth&Space	181	Mark Lombardi
R03	W	12:00pm-12:53pm	Lgt Engr Lab	154	Taras Kolomatski
R04	Tu	11:30am-12:23pm	Earth&Space	181	Siqing Zhang
R05	Tu	8:30am- 9:23am	Mathematics	P131	Yinzhe Gao
R06	M	11:00am-11:53am	Mathematics	P131	Owen Mireles Briones
R07	W	2:30pm- 3:23pm	Physics	P127	Taras Kolomatski
R20	M	12:00pm-12:53pm	Physics	P116	Mohamed El Alami
R21	F	12:00pm-12:53pm	Physics	P116	Ajmain Yamin
R22	W	10:00am-10:53am	Harriman Hll	112	Yinzhe Gao
R23	Tu	11:30am-12:23pm	Chemistry	128	Prithviraj Chowdhury
R24	F	9:00am- 9:53am	Mathematics	P131	Qianyu Chen
R29	M	10:00am-10:53am	Earth&Space	183	Owen Mireles Briones
R31	F	11:00am-11:53am	Mathematics	P131	Qianyu Chen
R32	M	2:30pm- 3:23pm	Earth&Space	183	Mohamed El Alami
R33	Tu	2:30pm- 3:23pm	Earth&Space	69	Mark Lombardi
R34	W	5:30pm- 6:23pm	Earth&Space	69	Yao Xiao
R35	Th	5:30pm- 6:23pm	Frey Hall	217	Siqing Zhang
LEC 30	FLEX	1:00am- 1:00am	Online		Deb Wertz
R30	FLEX	1:00am- 1:00am	Online		Deb Wertz

SOLUTIONS P2

Directions: Answer all questions in the space provided. You may use the blank backs of pages for scrap. No other paper is permitted. Show ALL relevant work. Calculators are not to be used. Circle your final answers. Unless instructed otherwise, you can use the derivative rules where needed.

1. Let $f(x) = \begin{cases} x \text{ for } 0 \le x \le 2 \\ -x+3 \text{ for } 2 < x < 5 \end{cases}$.



a) Sketch the graph on the given axes. Show the scale you're using.

b) Find its *domain* and *range*.

DOMAIN: $0 \le x < 5$ OR $[0, 5)$

RANGE: $-2 < x \le 2$ OR $(-2, 2]$

c) Find the following limits if they exist. If a limit doesn't exist write DNE.

i) $\lim_{x \to 2^-} f(x)$ = 2

ii) $\lim_{x \to 2^+} f(x)$ 1

iii) $\lim_{x \to 2} f(x)$ DNE

d) Find $f(2)$ 2

[15 points]

2. For the function shown at the right find the following. Use ∞ *or* $-\infty$ if needed. [10 points]

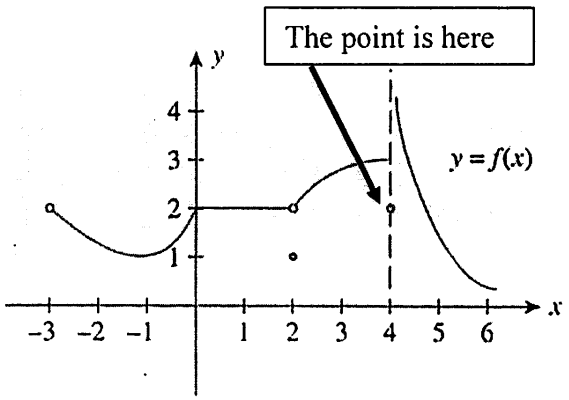
a) $\lim_{x \to 2^+} f(x)$ 2

b) $\lim_{x \to 2^-} f(x)$ 2

c) $\lim_{x \to 2} f(x)$ 2

d) Is f continuous at x = 4 ND

e) $\lim_{x \to 4^+} f(x)$ $+\infty$



SOLUTIONS P3

3. Given the following limits: $\lim_{x\to 1} f(x) = 4$, $\lim_{x\to 1} g(x) = -1$ and $\lim_{x\to 1} h(x) = 2$ [10 points]

Find the following.

a) $\lim_{x\to 1}(f(x) - (h(x)^3) + 3)$ $= \lim_{x\to 1} f(x) - \left(\lim_{x\to 1} h(x)\right)^3 + 3$

$= 4 - 2^3 + 3 = \boxed{-1}$

b) $\lim_{x\to 1} \frac{g(x)+1}{h(x)}$

$= \frac{\lim_{x\to 1} g(x) + 1}{\lim_{x\to 1} h(x)} = \frac{-1+1}{2} = \frac{0}{2} = \boxed{0}$

4. Find the following limits. Use ∞ *or* $-\infty$ for infinite limits. If there is no limit write DNE. [20 points]

a) $\lim_{x\to 9} \frac{x-9}{\sqrt{x}-3}\left(\frac{\sqrt{x}+3}{\sqrt{x}+3}\right) = \lim_{x\to 9} \frac{(x-9)(\sqrt{x}+3)}{(x-9)} = \lim_{x\to 9}(\sqrt{x}+3) = 3+3 = \boxed{6}$

b) $\lim_{x\to 1} \frac{x^2+x-2}{x-1} = \lim_{x\to 1} \frac{(x-1)(x+2)}{x-1} = \lim_{x\to 1}(x+2) = \boxed{3}$

c) $\lim_{x\to\infty} \frac{2x^3+3x-2}{3x^3-1}\left(\frac{1/x^3}{1/x^3}\right) = \lim_{x\to\infty} \frac{2 + 3/x^2 - 2/x^3}{3 - 1/x^3} = \frac{2+0-0}{3-0} = \boxed{\frac{2}{3}}$

d) $\lim_{x\to\infty} \frac{2x^3+3x-2}{3x^2-1}\left(\frac{1/x^2}{1/x^2}\right) = \lim_{x\to\infty} \frac{2x + 3/x - 2/x^2}{3 - 1/x^2} = \lim_{x\to\infty} \frac{2x+0-0}{3-0}$

$= \lim_{x\to\infty} \frac{2x}{3} = \boxed{+\infty}$

e) $\lim_{x\to\infty} \frac{2x^2+3x-2}{3x^3-1}\left(\frac{1/x^3}{1/x^3}\right) = \lim_{x\to\infty} \frac{2/x + 3/x^2 - 2/x^3}{3 - 1/x^3} = \frac{0+0-0}{3-0} = \frac{0}{3} = \boxed{0}$

4. (Continued)

f) $\lim_{x\to 1} \cos\left(\frac{1}{x-1}\right)$ AS $x\to 1$, $\frac{1}{x-1} \to \pm\infty$. $\lim_{x\to\infty} \cos(x)$ DNE

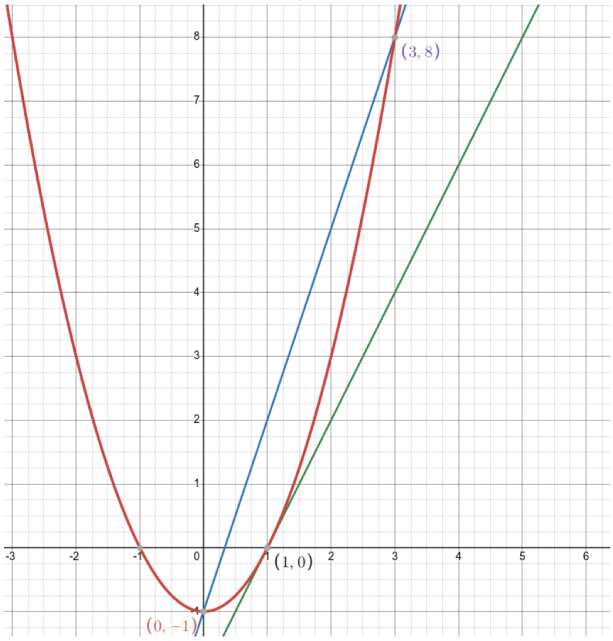
SO **LIMIT DNE**

g) $\lim_{x\to 2} \frac{2}{x^3-8}$ **DNE** SINCE $\lim_{x\uparrow 2^-} \frac{2}{x^3-8} = -\infty \neq \lim_{x\downarrow 2^+} \frac{2}{x^3-8} = +\infty$

h) $\lim_{x\to 0}\left(\frac{\sin x}{x}+4\right) = \left(\lim_{x\to 0}\frac{\sin x}{x}\right)+4 = 1+4 = \boxed{5}$

5. Given $g(x) = x^2 - 1$. [15 points]

a) Sketch the graph of g(x) on the given axes. Show the scale you're using.



b) Find the slope of the *secant line* passing through the points on the graph of g(x) for which x = 0 and x = 3.

$m = \frac{8-0}{3-1} = 4$

c) Find the slope of the *tangent line* passing through the point for which x = 1. Use any method you wish.

$\lim_{x\to 1} \frac{(x^2-1)-f(1)}{x-1} = \lim_{x\to 1} \frac{x^2-1}{x-1}$

$= \lim_{x\to 1} (x+1) = \boxed{2}$

d) Find the equation of the *tangent line* passing through the point for which x =1 and sketch it on the graph of g(x).

TANGENT LINE PASSES THRU (1,0) WITH SLOPE 2.

6. Consider the function $f(x) = x^3 - 5x^2 + 6x - 1$. Use the Intermediate Value Theorem to determine whether $f(x)$ takes on the value of 0 on the interval [1, 2]. Show any necessary work and give a reason for your answer. [10 points]

NOTE THAT $f(x)$ IS CONTINUOUS FOR ALL x, SINCE IT IS A POLYNOMIAL.

$f(1) = 1 - 5 + 6 - 1 = 1$ WHICH IS > 0.

$f(2) = 8 - 20 + 12 - 1 = -1 < 0$.

SINCE BY THE INTERMEDIATE VALUE THEOREM, f TAKES ON ALL VALUES BETWEEN $+1$ AND -1 ON $[1, 2]$, THERE IS A ZERO.

7. Use the definition of the derivative, $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ to find the derivative of $f(x) = 2x^2 + 3x + 5$. Show all work. Feel free to check by using the derivative rules but you must use the definition to get credit. [10 points]

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{2(x+h)^2 + 3(x+h) + 5 - (2x^2+3x+5)}{h}$$

$$= \lim_{h\to 0} \frac{2x^2 + 4xh + 2h^2 + 3x + 3h + 5 - 2x^2 - 3x - 5}{h}$$

$$= \lim_{h\to 0} \frac{4xh + 2h^2 + 3h}{h} = \lim_{h\to 0} (4x + 2h + 3) = \boxed{4x + 3}$$

8. If $2 - x^2 \le g(x) \le -2x + 3$ for all x find $\lim_{x\to 1} g(x)$ [10 points]

USING THE SQUEEZE THEOREM, WE HAVE

$$\lim_{x\to 1} (2 - x^2) \le \lim_{x\to 1} g(x) \le \lim_{x\to 1} (-2x + 3)$$

SO $$1 \le \lim_{x\to 1} g(x) \le 1$$

THUS $\boxed{\lim_{x\to 1} g(x) = 1}$