BERNHARD

MAT125 Spring 2014 Midterm #1

Your name:_____

TA's name:_____

Problem #1: Evaluate the following limits:

a) $\lim_{x\to 7}\frac{x^2-2x-35}{2x^2-11x-21}$ $= \lim_{x\to 7}\frac{(x-7)(x-5)}{(x-7)(2x+3)} = \lim_{x\to 7}\frac{x-5}{2x+3}$

$= \frac{7-5}{14+3} = \boxed{\frac{2}{17}}$

b) $\lim_{x\to -5}\frac{\frac{1}{5}+\frac{1}{x}}{5+x}$ $= \lim_{x\to -5}\frac{(x+5)5x}{5+x} = \lim_{x\to -5}\frac{1}{5x} = \boxed{\frac{-1}{25}}$

c) $\lim_{x\to 3}\frac{\sqrt{3x-5}-2}{x-3}$ $= \lim_{x\to 3}\left(\frac{\sqrt{3x-5}+2}{\sqrt{3x+5}+2}\right)\left(\frac{\sqrt{3x-5}-2}{x-3}\right)$

$= \lim_{x\to 3}\frac{(3x-5)-4}{(\sqrt{3x+5}+2)(x-3)} = \lim_{x\to 3}\frac{3x-9}{(x-3)(\sqrt{3x+5}+2)}$

$= \lim_{x\to 3}\frac{3}{\sqrt{3x+5}+2} = \lim_{x\to 3}\frac{3}{\sqrt{9-5}+2}$

$= \lim_{x\to 3}\frac{3}{\sqrt{4}+2} = \boxed{\frac{3}{4}}$

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Problem #2: Evaluate the following limits:

a) $\lim_{x\to\infty} \frac{\cos x}{x^2}$ NOTE COSX AND x^2 ARE DEFINED FOR ALL x, AND SINCE $x\to\infty$, WE CAN ASSUME $x>0$.

FOR ANY x $\quad -1 \le \cos x \le 1$

SINCE $x\neq 0$, $\quad -\frac{1}{x^2} \le \frac{\cos x}{x^2} \le \frac{1}{x^2}$

SO $\quad \lim_{x\to\infty}\left(\frac{-1}{x^2}\right) \le \lim_{x\to\infty}\frac{\cos x}{x^2} \le \lim_{x\to\infty}\left(\frac{1}{x^2}\right)$

$0 \le \lim_{x\to\infty}\frac{\cos x}{x^2} \le 0$, so $\boxed{\lim_{x\to\infty}\frac{\cos x}{x^2} = 0}$

b) $\lim_{x\to\infty} \frac{4x^3-5x+1}{7x^4+3x^2-x}\left(\frac{1/x^4}{1/x^4}\right)$

$$= \lim_{x\to\infty} \frac{4/x - 5/x^3 + 1/x^4}{7 + 3/x^2 - x/x^3} = \frac{0-0+0}{7+0-0} = \boxed{0}$$

c) $\lim_{x\to\infty} \frac{e^{-x}+5}{e^{-2x}+4}$ (RECALL THAT $\lim_{x\to\infty} e^{-x} = 0$.)

$$= \frac{\lim_{x\to\infty} e^{-x}+5}{\lim_{x\to\infty} e^{-2x}+4} = \frac{0+5}{0+4} = \boxed{\frac{5}{4}}$$

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Problem #3: Use the definition of the derivative to find $f'(x)$.

$$f(x) = 3x^2 - 5x + 4$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\left[3(x+h)^2 - 5(x+h) + 4\right] - \left[3x^2 - 5x + 4\right]}{h}$$

$$= \lim_{h \to 0} \frac{3x^2 + 6xh + h^2 - 5x - 5h + 4 - 3x^2 + 5x - 4}{h}$$

$$= \lim_{h \to 0} \frac{6xh + h^2 - 5h}{h}$$

$$= \lim_{h \to 0} (6x + h - 5) = 6x - 5.$$

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Problem #4: Use the definition of the derivative to find $f'(x)$ at $x=2$.

$$f(x)=\frac{12}{3x-1}$$

$$f'(2)=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$

$$=\lim_{h\to 0}\frac{\frac{12}{3(2+h)-1}-\frac{12}{5}}{h}=$$

$$=\lim_{h\to 0}\frac{-36h/(25+15h)}{h}=\lim_{h\to 0}\frac{-36}{25+15h}$$

$$=\boxed{\frac{-36}{15}}$$

OR YOU COULD DO $\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}$

BUT IT IS EQUALLY TEDIOUS.