Chapter 1

Preliminaries about Maple

Maple is a comprehensive general purpose computer algebra system which can do symbolic and numerical calculations and has facilities for 2 and 3 dimensional graphical output. The sequence of calculus courses maybe structured so that Maple can be used as a tool that will help you gain a complete understanding of the material discussed. You will discover that the capability of Maple goes beyond the realm of calculus; it is a tool that is used more and more in education and scientific research in mathematics and engineering.

This chapter is a brief introduction to the use of Maple[1] in the environment that you will discover at Stony Brook. It is not exhaustive and its sole purpose is to guide you through the first steps needed to start using this tool. Once you become familiar with the basics of Maple and can appreciate its advantages, we encourage you to experiment with it, not only to learn mathematics in more detail but also to help you in other engineering courses and your own research. More advanced documentation is available on line, in the computer lab, and in the library.

As a student at the University, you will be given a computer account. In order to use it, you will need to keep in mind your login name and your *password*, a secret code that is used to identify and validate your right to access the computer system. This introduction includes instructions on how to get into the system and open a Maple session.

Do not reveal your password to anybody. Choose a password which is easy to remember without writing it down. Select a mixture of upper and lower case letters, numeric and punctuation characters, and spaces between them. Avoid passwords which are words in the dictionary.

[1] We should point out that the material in this chapter was originally written with release 3 of Maple in mind. You may find a few small differences with the current release which we have tried to mark in the text. These differences are extremely minor, and should be pointed out by your instructor. The later chapters in these notes are all tested with release 5 of maple, which is current as of 1998.

1.1 Logging in and opening a Maple session

When looking at the screen of any terminal in the computer lab, you will see a `login name` window in the center of the screen:

- Type in your login name and press the `return` key.
- Type in your password and press the `return` key.

To *open a Maple session* procceed as follows:

- Position the cursor in the background (off the window's area) and hold down the left button of the mouse. A `Root Menu` will be displayed.
- While still holding down the left button of the mouse, move the cursor down the `Root Menu` until you position it on the `Tools` option, and then move it a little bit to the right until you see a submenu displayed.
- Keep the left button down and move the cursor along the `Tools` submenu until it is placed on the `Maple` option. Release the left button now.

After a few seconds, a Maple window will appear on the screen. It will show a standard Maple message with some information about the software you are about to use, Maple V Release 4.

The Maple window is ready to accept your commands when the Maple prompt `>` shows on it. We will illustrate the basic use of Maple through examples assuming that you are in this input mode. Maple will record all of your commands on a worksheet. Later on, we shall tell you how to save the worksheet in case you want to use its results on another occasion.

1.2 Basic Maple

You can use Maple for numerical calculations, for symbolic manipulations, and for graphing purposes. *All Maple commands must be ended with a semicolon* `;` *and the corresponding stroke of the return key.* To familiarize yourself with it, try to run the following examples and analyze the results. The commands that you type in and the Maple results are shown here in a way that is similar to what you see on the screen.

1.2.1 Maple for numeric calculations

Most of the time you will be using Maple as a calculator. You can also write programs in Maple, but usually you will be using the tool expecting an immediate response. You type in your statement at the Maple prompt `>` and Maple responds with an answer:

```
> 1/2+3;
                                  7/2
```

The symbols for the basic operations are +, -, * and /. Exponentiation is denoted by ^:

```
> 77*(3^5+5^2)/(99-47);
                                 5159
                                 ----
                                  13
```

In the first example, the inverse of 2 is added to 3. In the second, we input $\frac{77(3^5+5^2)}{99-47}$, and Maple computes its value before printing the output.

You must always use the asterisk * for multiplication. Forgetting it in expressions like `sin(2*x)` or `4*x` is easy, and will result in a syntax error message.

Maple works primarily with whole numbers or fractions. However, you can force it to write the decimal expansion of any number with `evalf`:

```
> evalf(100*Pi);
                              314.1592654

> b:=sqrt(2);
                                   1/2
                               b := 2

> evalf(b);
                              1.414213562
```

Be careful with this example. The *assignment* of the variable `b` to $\sqrt{(}2)$ is done with := rather than with =. This will be the case of any other assignment operation you do in Maple, as we discuss below.

Notice the difference in the following two examples:

```
> sum(1/(2*i),i=1..10);
                                 7381
                                 ----
                                 5040

> sum(1.0/(2*i),i=1..10);
                              1.464484127
```

In both cases, we compute $\sum_{i=1}^{10}(2i)^{-1}$. The factor `1.0` in the second case indicates to Maple that you want to obtain the decimal expansion of the result.

1.2.2 Maple for symbolic manipulations

We can ask Maple to calculate the sum of the cube of the first n integers:

```
> sum(i^3,i=1..n);
                                4              3              2
                    1/4 (n + 1)  - 1/2 (n + 1)  + 1/4 (n + 1)
```

We can ask to factor the resulting polynomial in n:

```
> factor(");
                                       2        2
                                  1/4 n  (n + 1)
```

In this example " is used[2] to refer to the result previously obtained. Thus, in effect, the example above is equivalent to

```
> factor(1/4*(n + 1)^4 - 1/2*(n + 1)^3 + 1/4*(n + 1)^2);

                                       2        2
                                  1/4 n  (n + 1)
```

One can expand or factor conveniently:

```
> p:=(x+1)^4*(x-6)^3;
                                          4        3
                              p := (x + 1)  (x - 6)

> expand(");
           7       6       5        4        3        2
          x  - 14 x  + 42 x  + 112 x  - 287 x  - 882 x  - 756 x - 216

> factor(");
                                        4        3
                                  (x + 1)  (x - 6)
```

We can solve equations:

[2] If you have Maple V release 5 or later, you should use % to refer to the previous result instead.

```
> quadratic:=a*x^2+b*x+c;
                                        2
                          quadratic := a x  + b x + c

> solve(quadratic=0,x);
                          2         1/2              2         1/2
                  - b + (b  - 4 a c)         - b - (b  - 4 a c)
              1/2 ---------------------, 1/2 ---------------------
                            a                          a
```

Notice that `solve` provides both roots of the quadratic polynomial.

Maple also divides polynomials. Here the command you execute has a syntax which is a little bit more complicated. Consider the following example:

```
> p:=x^10-1;
                                       10
                                  p := x   - 1

> q:=x-1;
                                   q := x - 1

> divide(p,q,b);
                                      true
```

We assign the polynomials $x^{10}-1$ and $x-1$ to the variables `p` and `q`, respectively, and then ask Maple to divide `p` by `q`. Maple replies with the statement `true` to indicate that the polynomial $q = x - 1$ divides exactly the polynomial $p = x^{10} - 1$, and places the quotient in the third argument of the `divide` command. Thus, if you want to see the quotient all that you need to do is to see what the variable `b` stands for now:

```
> b;
                   9    8    7    6    5    4    3    2
                  x  + x  + x  + x  + x  + x  + x  + x  + x + 1
```

Of course, the answer produced by `divide` will be `false` in the case when the polynomials do not divide each other exactly. In that situation, one can use the commands `quo` and `rem` to find the *quotient* and *remainder* of the division:

```
> r:=x^2+2*x+3;
                                        2
                                  r := x  + 2 x + 3
> divide(p,r,c);
                                      false
```

```
> quo(p,r,x);
            8      7    6      5       4       3       2
           x  - 2 x  + x  + 4 x  - 11 x  + 10 x  + 13 x  - 56 x + 73

> rem(p,r,x);
                                    - 220 + 22 x
```

We would have obtained an error if we would have tried `divide(p,r,b)`, trying to place the quotient of the division in the variable `b`: it is no longer a variable since it has a value from the previous example. Had we wanted to proceed that way, it would have been necessary to *unassign* the value of `b` first:

```
> b:='b';
                                        b := b
```

Further discussion on this is given in section 1.3.2.

We can use `solve` for systems of equations. For example:

```
> eq1 := 1*x+(1/2)*y+(1/6)*z =1;

                           eq1 := x + 1/2 y + 1/6 z = 1

> eq2 := (1/2)*x+(1/6)*y+(1/24)*z =0;

                         eq2 := 1/2 x + 1/6 y + 1/24 z = 1

> eq3 := (1/6)*x+(1/24)*y+(1/120)*z =1;

                       eq3 := 1/6 x + 1/24 y + 1/120 z = 1

> solve({eq1,eq2,eq3},{x,y,z});

                           {x = 39, y = -216, z = 420}
```

Maple also solves inequalities:

```
> solve(x^2-4*x-7>0,x);
                                     1/2           1/2
                        {x < 2 - 11   }, {2 + 11    < x}

> solve(abs(1-x^2)<1/2,x);
                 1/2                  1/2         1/2                1/2
         {- 1/2 6    < x, x < - 1/2 2   }, {1/2 2    < x, x < 1/2 6   }
```

1.2.3 Graphing in Maple

Let us now use the graphics facilities of Maple:

```
> f:=x->sin(x^2);
```

$$f := x \to \sin(x^2)$$

```
> plot({f(x)}, x=-3..3);
```

A new window is created displaying the graph of the function $sin(x^2)$ as indicated in Fig. 1.

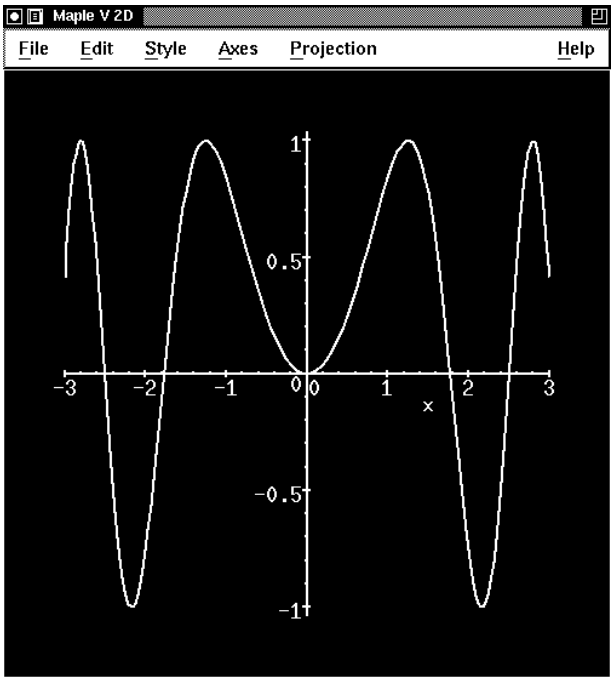

Figure 1.2.1: Window created by Maple with the plot of $sin(x^2)$ on $-3 \leq x \leq 3$

Notice the menu on top of the window displaying the graph. Clicking on `File` and selecting the `Exit` option from it, or pressing the `q` key while the cursor is on this window, makes the window go away.

In section 1.8 we shall discuss in further detail the graphing features of Maple.

1.2.4 On line Maple help

There are two kinds of help available on line in Maple. The first is useful if you know the name of the command you need to use and want to find out details on its syntax and how to use it properly. For example, to get information on Maple's `factor` command we execute

```
> ?factor;
```

Maple creates another window that very likely will contain more information than you really need or care about. You can read all or part of the information displayed in this window by moving the text down with the `scroll bar` to the right of the window. To do this, place the cursor on the `scroll bar` and move the mouse down (or up).

You should notice that at the end of all of this help windows you will find a good set of examples on the uses of the command you are inquiring about. This examples can be copied and pasted in your worksheet and executed. In this way, you can see what the command really does.

Even if the word you input is not quite a Maple command but something close, Maple will suggest relevant alternatives to try:

```
>?plotting;
Try one of the following topics:
           {plotsetup, plot, plot3d, plots, plotdevice, PLOT, PLOT3D}
```

This is an excellent way to learn about new commands.

The other way to find help about Maple is to use the `Help Browser`. Click on the `help` menu located on the right side of the top of the Maple window, and select from it the browser option. Try experimenting with it and with the other options available in this `help` menu.

1.3 Assignments, Functions and Constants

Let us discuss the syntax of some basic Maple commands, as well as some functions and constants that Maple knows.

1.3.1 Assignment statements

The assignment is the most basic and used of Maple statements. It allows you to give names to expressions you want the computer to remember for later use. It has the *colon-equals* format `name := expression`. The thing on the left side of `:=` is a *name*, i.e. a sequence of letters and digits that begins with a letter. Certain names must be avoided because Maple uses them for special purposes (see section 1.3.5):

```
> a:=3; b:=5; c:=22/7;
                                        a := 3

                                        b := 5

                                      c := 22/7
```

Now you can use a, b and c as though they are numbers. Thus, we have

```
> a+b;
                                  8
> c*a;
                                66/7
> a-b^2;
                                -22
```

1.3.2 Variables and subsequent assignments: unassignments

If you type a variable name and a semicolon, Maple will tell you what the variable stands for:

```
> c;
                                22/7
> x;
                                 x
```

If you specify y as a function of x and then assign to x a particular value, this value of x gets substituted into y:

```
> y:=sqrt(x+3);
                                      1/2
                           y := (x + 3)
> x:=sin(w);
                             x := sin(w)
> y;
                                      1/2
                           (sin(w) + 3)
```

If x gets changed, then y will also change. Maple remembers the original definition of y:

```
> x:=cos(t);
                             x := cos(t)
> y;
                                      1/2
                           (cos(t) + 3)
```

Notice that Maple evaluates x before telling you what the variable y stands for now. To undo this kind of thing, you can *unassign* variables. To do this for x execute:

```
> x:='x';
                                x := x
> y;
                                     1/2
                              (x + 3)
```

1.3.3 Functions known to Maple. How to define your own functions

Maple has all the standard mathematical functions in its library. You can make use of them by their appropriate name. We list some of these.

- Trig and inverse trig functions: `sin`, `cos`, `tan`, `cot`, `sec`, `csc`, `arcsin`, `arccos`, `arctan`, `arccot`, `arcsec`, `arccsc`.
- Exponential and logarithm: Maple uses `exp(x)` for e^x and either `log(x)` or `ln(x)` for the natural logarithm $\ln x$. Logarithm in other bases are also available. For example, `log[10](x)` is $\log_{10} x$.
- Hyperbolic functions: `sinh`, `cosh`, etc.
- Square roots and absolute values: `sqrt(x)` and `abs(x)`.

Of course, you will find yourself in a situation where you would like to define your own functions. Suppose you are analyzing the expression $\sqrt{1+x^2}$, plugging several values into it and calculating the result. Maple does not have a built-in function to do this, but you can define one yourself:

```
> f:=x->sqrt(1+x^2);
```

$$f := x \to \sqrt{1 + x^2}$$

Carefully notice the syntax. `f` is the function written to the right side of `:=`, and sends the variable x to $\sqrt{1+x^2}$. The arrow `->` is made with the minus and greater keys.

After such an assignment, you can use the name `f` in the same way you use other names of functions known to Maple, e.g. `exp` or `sin`:

```
> f(0); f(1); f(-1); f(3*x+1);
```

$$1$$

$$2^{1/2}$$

$$2^{1/2}$$

$$(2 + 9\,x^2 + 6\,x)^{1/2}$$

We are so accustomed to the notation `f(x)` to refer to a function that we might accidentally use it in Maple.

Never try to define a function with an assignment like

```
> f(x):=sqrt(x-2);
                                        1/2
                              f(x) := (x-2)
```

Maple will let you do it, but the resulting `f(x)` is useless. You will not be able to substitute into it or do anything you normally do with functions.

Maple uses parentheses `( )` to group expressions together in a convenient form as in

```
> 2*(x+4)^3;
                                          3
                                 2 (x + 4)
```

or to *delimit the arguments of a function.* In handwritten mathematics, we sometimes cheat and write $\sin x$ instead of $\sin(x)$. In Maple, this is not permissible: you must always write `sin(x)`. Nested parentheses are permitted:

```
> f(x);
                                       2 1/2
                                (1 + x )
> sin(sqrt(x+2)+ln(abs(x)));
                                   1/2
                        sin((x + 2)    + ln(abs(x)))
```

1.3.4 Special constants

Maple knows several constants. To find them out, execute the command

```
> constants;
                  false, gamma, infinity, true, Catalan, E, Pi
```

The names of these constants stand for `false`, γ, ∞, `true`, `Catalan`, e and π, respectively.[3] Be careful! Maple is case sensitive. Therefore `pi` is not the same as `Pi`. For example,

```
> cos(Pi/4);
                                        1/2
                                   1/2 2
> cos(pi);
                                   cos(pi)
```

[3] As of release 5 of Maple V, `E` no longer represents the constant $e \approx 2.71828\ldots$. Use `exp(1)` instead.

1.3.5 Maple reserved names

There are many commands and special objects in Maple that have preassigned names. In creating your own expressions and setting the name of your own variables, you should never use these particular names. They are:

```
and    by     do        done   elif    else   end    fi    for     from
if     in     intersect local  minus   mod    not    od    option  options
or     proc   quit      read   save    stop   then   to    union   while
```

The following names can be used, but it is not a good idea to do so:

```
cos  E    equation  exp   I   list  ln   log   matrix  sin   string  tan  vector
```

If you would like to use a particular name but are not certain if Maple gives it a special meaning, execute the command

```
> ?name;
```

where `name` is the name you want to check. If you get a Maple help screen for the name, it is probably safer to avoid it. You should only insist on using the name when the Maple response to the command above is something as in the following case:

```
> ?weirdname;
  The argument passed to help was:   weirdname
  Sorry, unable to explain this topic.  See  ?index  for
  a list of topics which are available.
```

1.4 The limit command

The limit command is used to compute limits. Its syntax is basically

```
> limit(f,x=a);
```

where `f` is a Maple algebraic expression which (in general) depends on the variable `x` and `a` is the expression which `x` approaches. For example, to compute

$$\lim_{x\to 2} \frac{3x-6}{x^2-4},$$

we execute the Maple command

```
> limit((3*x-6)/(x^2-4),x=2);
                                3/4
```

We can also try $\lim_{x\to 0} \frac{1}{x}$:

```
> limit(1/x,x=0);
                                  undefined
```

In the following exercise, we define the slope of a straight line passing through the points (x_1, y_1), (x_2, y_2), use this function to find the slope $m(x)$ of the line passing through the points $(1, 1)$ and (x, x^2), and finally compute the limit of $m(x)$ as $x \to 1$:

```
> slope:=(x1,y1,x2,y2)->(y2-y1)/(x2-x1);
                                                 y2 - y1
                      slope := (x1,y1,x2,y2) -> -------
                                                 x2 - x1
> m:=x->slope(1,1,x,x^2);
                                                    2
                         m := x -> slope(1, 1, x, x )

> limit(m(x),x=1);
                                       2
```

Maple is sometimes unable to determine a limit or whether it exists. In such a case, it will return nothing after you execute the limit command.

Maple computes *two-sided* limits. For example, if you specify the argument `x=1` as in the previous example, Maple assumes you mean that `x` approaches `1` from either the right or the left through real values only (as opposed to complex ones). However, Maple can compute one sided and complex limits also:

```
> limit(1/x,x=0,right);
                                    infinity

> limit(1/x,x=0,left);
                                   - infinity

> limit(x*log(x),x=0,complex);
                                       0
```

You can also compute limits as $x \to \infty$ and limits of functions of more than one variable:

```
> limit(arctan(x),x=infinity);
                                     1/2 Pi

> limit(x/(x^2+y^2),{x=0,y=0});
                                  undefined
```

A common error is to try to compute the limit of a function `f(x)` when `x` has been previously given a value. If you find yourself in this situation, unassign the value of `x` executing the command `x:='x';`.

1.5 The diff (and Diff) command

The `diff` command is used to compute derivatives of Maple expressions. Its syntax is basically

```
diff(f,x);
```

where `f` is an algebraic expression and `x` is the variable with respect to which the derivative is taken. For example, to compute

$$\frac{d}{dx}\frac{3x-6}{x^2-4},$$

we execute the Maple command

```
> diff((3*x-6)/(x^2-4),x);
```

$$\frac{3}{x^2-4} - 2\,\frac{(3x-6)\,x}{(x^2-4)^2}$$

You must specify the variable with respect to which you wish to take the derivative because, almost all the time, there are constants and other parameters around:

```
> diff(exp(a*x),x);
```

$$a\,\exp(a\,x)$$

Maple knows theoretical results about derivatives. For example, even if the functions `g(x)` and `h(x)` are not defined, Maple will give you the derivative of their product:

```
> diff(h(x)*g(x),x);
```

$$\left(\frac{d}{dx}h(x)\right) g(x) + h(x)\left(\frac{d}{dx}g(x)\right)$$

1.5.1 Higher order derivatives

To compute higher order derivatives, you can of course iterate the above command:

```
> diff(diff(3*sin(x),x),x);
```

$$-3\sin(x)$$

Such a command first calculates the derivative with respect to `x` of the function `3*sin(x)` and then differentiates the result to obtain the second order derivative of the original function. The same result can be accomplished with either one of the following commands:

```
> diff(3*sin(x),x,x);
                                  - 3 sin(x)

> diff(3*sin(x),x$2);
                                  - 3 sin(x)
```

There is an obvious extension of this command when calculating more than two derivatives. You can also calculate partial derivatives of higher order:

```
> diff(ln(x)*exp(-2*x),x$3);
          exp(- 2 x)     exp(- 2 x)      exp(- 2 x)
        2 ---------- + 6 ---------- + 12 ---------- - 8 ln(x) exp(- 2 x)
               3              2               x
              x              x

> diff(exp(x)*y^2*sin(z),x,y,z);
                               2 exp(x) y cos(z)
```

1.5.2 Implicit differentiation

Maple knows how to take derivative of both sides of an equation, and, as indicated above, Maple can also take derivatives "theoretically" even if the definition of the functions involved are not specified. However, you must always indicate *explicitly* the dependence of the functions on their variables. So, if you are thinking of `y` as a function of `x`, you must write `y(x)` in your equation rather than just `y`.

Here is a typical implicit differentiation problem. Consider the equation $x^2y - 3y^3x = 0$. Find the slope of the graph of the curve defined by this equation at the point $(3, 1)$:

```
> eq:=x^2*y(x)-3*y(x)^3*x = 0;
                                 2              3
                          eq := x  y(x) - 3 y(x)  x = 0

> deq:=diff(eq,x);
                         2 /  d      \          2   /  d      \          3
     deq := 2 x y(x) + x  |---- y(x)| - 9 y(x)  x |---- y(x)| - 3 y(x)  = 0
                          \ dx      /             \ dx      /

> solve(deq,diff(y(x),x));
```

```
                                    3
                  2 x y(x) - 3 y(x)
                - ------------------
                     2          2
                    x  - 9 y(x)  x
```

```
> subs(y(x)=1,x=3,");
                                1/6
```

The first command defines the equation relating `x` and `y`, the second defines an equation `deq` giving the relation between `x`, `y(x)` and the derivative of `y` with respect to `x`. Using `solve`, we solve `deq` to find the derivative as a function of `x` and `y(x)`, and in the result we substitute `x=3` and `y=1` to obtain the desired slope (see section 1.7 to learn about the `subs` command).

Again, the major caveat with the Maple derivative command is to avoid calculating derivatives with respect to a variable that has been previously given a value. In such an instance, unassign the value of the variable first.

Occasionally, to make your worksheets easier to read, you may wish to have Maple display a derivative in standard mathematical notation without evaluating it. For this purpose there is an *inert* capitalized form of the `diff` command: `Diff`. The two forms `diff` and `Diff` are usually combined to produce meaningful sentences:

```
> Diff(exp(x)/(1-x),x);
```

$$\frac{\partial}{\partial x}\frac{e^x}{1-x}$$

```
> Diff(exp(x)/(1-x),x)=diff(exp(x)/(1-x),x);
```

$$\frac{\partial}{\partial x}\frac{e^x}{1-x} = \frac{e^x}{1-x} + \frac{e^x}{(1-x)^2}$$

This command is particularly useful to produce easy to read worksheets that involve partial derivatives:

```
> f:=x^3*exp(y)-sin(x*y);
                                  3
                      f := x  exp(y) - sin(x y)
```

```
> Diff(f,x,x,y)=diff(f,x,x,y);
```

$$\frac{\partial^3}{\partial y \partial x^2} x^3 e^y - \sin(xy) = 6xe^y + \cos(xy)xy^2 + 2\sin(xy)y$$

1.6 The int (and Int) command

The `int` command is used to compute both definite and indefinite integrals of Maple expressions. Its syntax is basically

```
int(f,x);
```

where `f` is an algebraic expression and `x` is the integration variable. For example, to compute

$$\int \frac{3x-6}{x^2-4}\,dx\,,$$

we execute the Maple command

```
> int((3*x-6)/(x^2-4),x);
                                3 ln(x + 2)
```

Notice that Maple does not provide the constant of integration. You will occasionally have to take this into account and provide your own constant.

You must specify the variable of integration. In expressions involving other parameters, Maple assumes that you want the integral of the expression as the variable you specify changes and that all other parameters in the expression represent constants:

```
> int(exp(a*x),x);
                                  exp(a x)
                                  --------
                                      a
```

To compute a definite integral, the *range* over which the integration variable moves must be specified:

```
> int(x^2*exp(x),x=0..2);
                                2 exp(2) - 2
```

The `int` command in Maple has a particular behavior in certain situations:

- If Maple cannot compute the integral in *closed form*, it will return it unevaluated:

```
> int(ln(sin(sqrt(x^12-5*x^7+50*x+2))),x);
                      /
                     |           12      7            1/2
                     |  ln(sin((x   - 5 x  + 50 x + 2)   )) dx
                     |
                    /
```

- Sometimes the integral cannot be evaluated in closed form in terms of *elementary* functions, but the answer has a special name in mathematical circles due to its importance in applications. For example,

```
> int(sin(2*x)/x,x);
                                    Si(2 x)
```

  Here `Si` is the *special name* of one of these functions which appear frequently in mathematical physics. To learn more about it, execute the command

```
> ?Si;
```

  which will bring up a window with information about the function `Si`. If Maple responds to an integral with one of these functions, it is quite likely that the integral cannot be evaluated in terms of elementary functions.

- Very complicated answers contain the `%n` notation:

```
> int(1/(x^8 +1),x);
                                 -----
                                  \
                                   )     _R ln(x + 8 _R)
                                  /
                                 -----
                                _R = %1

                                                      8
%1 :=                       RootOf(1 + 16777216 _Z )
```

  In this answer, the sum is taken over all roots R of the polynomial $1 + 16777216z^8$, and the summand is $R \log(x + 8R)$.

1.6.1 Numerical integration

With the `evalf` command, you can force Maple to apply a numerical approximation technique for definite integration:

```
> evalf(Int(sqrt(1+x^10),x=0..1));
                                  1.040899075
```

Notice that the command for integration is *capitalized*. This prevents Maple from attempting to evaluate the integral symbolically and then just `evalf`ing the answer.

1.6.2 Approximations through Riemann Sums

Maple also computes expressions that approximate a definite integral using rectangles, trapezoids, etc. It can also do approximations using Simpson's rule. The only difficulty is that these commands are contained in a separate library called `student` and you must load this library into computer memory before you can use it:

```
> with(student):
 [D, Int, Limit, Sum, changevar, combine, completesquare, distance, intercept,

     intparts, isolate, leftbox, leftsum, makeproc, maximize, middlesum,

     midpoint, minimize, powsubs, rightbox, rightsum, showtangent, simpson,

     slope, trapezoid, value]
```

Once loaded, you can play with it. For example, if you want to use the left sum approximation to an integral, the height of each rectangle is determined by the value of the function at the left side of each interval. You may specify the number of intervals you wish to use. If you do not, Maple will use four intervals by default:

```
> leftsum(x^4*ln(x),x=1..4,10);
```

$$\sum_{i=0}^{9} \frac{3}{10}\left(1+\frac{3}{10}i\right)^4 \ln\left(1+\frac{3}{10}i\right)$$

Maple can also draw pictures of the rectangles used to approximate the integral of a function on a given interval:

```
> leftbox(1/x,x=1..2,10);
```

and the resulting figure is displayed in Fig. 2.

The following is the result of using the Simpson's rule for the function $xsin(x^2)$ on the interval $1 \leq x \leq 5$. Since the number of intervals is not specified, Maple assumes four equal intervals by default:

```
> simpson(x*sin(x^2),x=1..5);
```

$$\sum_{i=0}^{2}$$

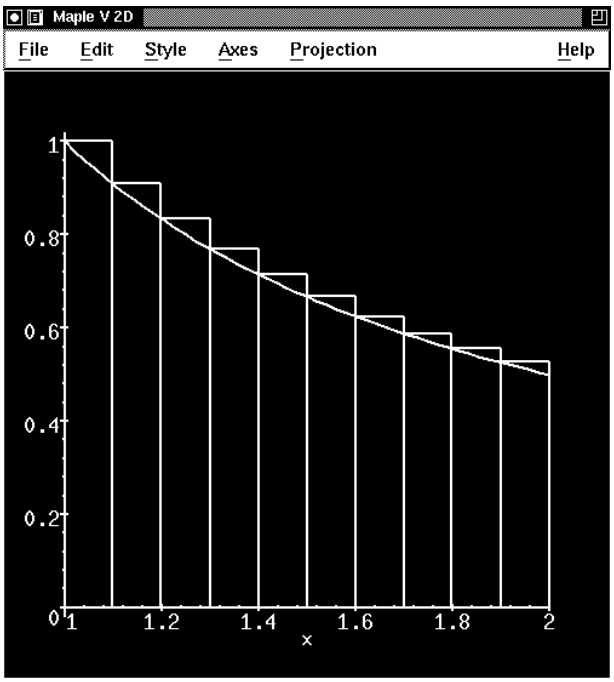

Figure 1.6.2: Left sum approximation of $\int_1^2 \frac{1}{x} dx$ with 10 rectangles

```
                                 |  )    2 i sin(4 i )|
1/3 sin(1) + 5/3 sin(25) + 4/3   | /                  |
                                 |-----               |
                                 \i = 1               /

          /  1                                \
          |-----                              |
          | \                              2  |
    + 2/3 |  )    (1 + 2 i) sin((1 + 2 i) )  |
          | /                                 |
          |-----                              |
          \i = 1                              /
```

1.6.3 Multiple integrals

Maple does iterated integrals:

```
> int(int(x^2*y^3,x=0..y),y=2..3);
```

$$\frac{2059}{21}$$

This is literally an iterated integral: the integral of x^2y^3 with respect to x on the interval $[0, y]$ is nested inside the integral with respect to y on the interval $[2, 3]$, in effect calculating

$$\int_2^3 \int_0^y x^2 y^3 dx dy \, .$$

As with `diff`, there is an *inert* form `Int` of the integral command which can be used in combination with `int` to produce easily readable worksheets:

```
> Int(ln(1+3*x), x=1..4);
```

$$\int_1^4 \ln(1+3x) dx$$

```
> Int(ln(1+3*x), x=1..4)=int(ln(1+3*x),x=1..4);
```

$$\int_1^4 \ln(1+3x) dx = \frac{13}{3}\ln(13) - 3 - \frac{4}{3}\ln(4)$$

A typical error is to try to integrate with respect to a variable that has been given a particular value you forgot about. Unassign the variable first.

1.7 The subs command

You have learned in the previous sections that if you assign a particular value to a variable, Maple will remember it for the rest of the session until you unassign it. At times, you might like to *plug-in* some values into an expression without altering the variable (or variables) involved. Maple has the `subs` command which reports the result of making such a substitution:

```
> y:=x+3;
```

```
y := x + 3
```

```
> subs(x=2,y);
```

```
5
```

Notice that Maple reports the value obtained when plugging in `x=2` into `y` without giving that value to `x` or altering `y` in any way:

```
> x;
```

```
x
```

```
> y;
```

```
x + 3
```

You can make more than one substitution:

```
> z:=x+sin(2*h):

> subs(x=3*u+1,h=4*Pi*q,z);
                              3 u + 1 + sin(8 Pi q)
```

In this example, the first assignment is ended with a colon : to suppress the display. You can do this whenever you want to make an assignment which you do not want to see displayed. Then, plugging $x = 3u + 1$ and $h = 4\pi q$ into the expression for z, we obtain $3u + 1 + \sin(8\pi q)$. Notice that the values of `x` and `h` that we plug in do not need to be enclosed in braces when issuing the command.

You should be aware of a few things when using the `subs` command:

- In certain situations, `subs` will not be able to find the expression you are substituting for if it is complicated. In those cases, try to *reword* your substitution request so that Maple *understands it*, or break the task into several manageable ones.

- You need not fear *circular substitutions*:

  ```
  > z;
                                      x + sin(2 h)

  > subs(x=2*x,z);
                                    2 x + sin(2 h)
  ```

  On the other hand, long chains of self-referential substitutions may produce results which are hard to predict:

  ```
  > subs(h=x,x=z,z);
                          x + sin(2 h) + sin(2 x + 2 sin(2 h))
  ```

  Simultaneous substitutions are made enclosing the values you plug in in braces. This is in contrast with the left-to-right substitution above:

  ```
  > subs({h=x,x=z},z);
                                 x + sin(2 h) + sin(2 x)
  ```

1.7.1 Why is subs useful?

Often it is better to use `subs` rather than to make assignments to the variables in an expression. For example, if

```
> y:=3*x^2+5*x+2;
```

$$y := 3x^2 + 5x + 2$$

and you want to compute the difference quotient $(y(x+h)-y(x))/h$, then you should execute

```
> dq:=(subs(x=x+h,y)-y)/h;
```

$$dq := \frac{3(x+h)^2 + 5h - 3x^2}{h}$$

because if we set

```
> x:=x+h;
```

```
Warning: Recursive definition of name
```

$$x := x + h$$

any call to `y` will result in

```
> y;
Error, STACK OVERFLOW
```

and quite likely, your Maple session will crash. On the other hand,

```
> x:=4;
```

$$x := 4$$

```
> y;
```

$$70$$

is allright, but now `y` is a number and we cannot plug any other values of `x` into it. `subs` is a better way to do the plugging in.

1.8 Plotting with Maple

Let us now use the graphics facilities of Maple:

```
> f:=x->x^2;
```

$$f := x \to x^2$$

```
> plot({f(x)}, x=-10..10);
```

```
>
```

A new window is created displaying the graph of the function x^2 from $x = -10$ to $x = 10$ as indicated in Fig. 3. Notice that the window has a menu which allows you to manipulate your graph or even get help with a click of the mouse. To close the window, place the cursor in the `File` menu and, holding down the left key of the mouse, select `close`.

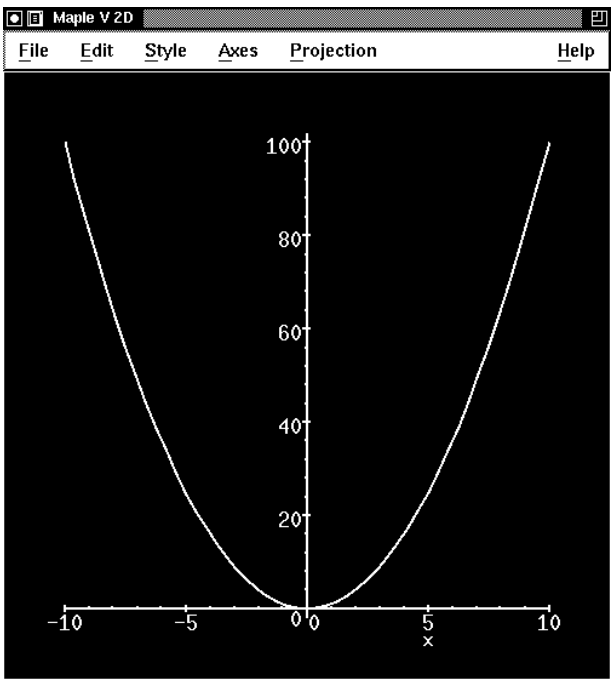

Figure 1.8.3: Window created by Maple with the plot of x^2 on $-10 \leq x \leq 10$

The basic syntax of the plot command is

```
> plot(f,range);
```

where `f` is the expression to be plotted and `range` is the range of the parameter(s) for which you would like to see the plot of `f`. In the example above, we indicated the range of `x` by `x=-10..10`. Maple automatically chooses a scale on the vertical axis. Observe what happens when executing the next two commands:

```
> plot(x^2-x,x=-2..2,y=-2..2);
```

```
> plot(x^2-x,x=-2..2,-2..2);
```

In both cases, the plot is displayed for values of `y` between -2 and 2, but in the example where the variable `y` is explicitly used in the command, the vertical axis has a label. The variable in the domain of the function *must always* be specified.

1.8.1 Plotting several functions (or curves) on the same axes

We can plot more than one function (or curve) on the same coordinate system. Consider the following example, recalling the slope function of the line passing through the points (x_1, y_1), (x_2, y_2), used to define the slope $m(x)$ of the line passing through $(1, 1)$ and (x, x^2):

```
> slope:=(x1,y1,x2,y2)->(y2-y1)/(x2-x1);
                                              y2 - y1
                      slope := (x1,y1,x2,y2) -> -------
                                              x2 - x1
> m:=x->slope(1,1,x,x^2);
                                                 2
                          m := x -> slope(1, 1, x, x )
```

We now define the equation of the lines passing through $(1, 1)$ and (x, x^2) for $x = 2, 3, 4, 5$ and plot the four lines together with the function `f(x)`. The result is shown in Fig. 4:

```
> line1:=subs(x=2,m(x))*(x-1)+1;
                                line1 := 3 x - 2

> line2:=subs(x=3,m(x))*(x-1)+1;
                                line2 := 4 x - 3

> line3:=subs(x=4,m(x))*(x-1)+1;
                                line3 := 5 x - 4

> line4:=subs(x=5,m(x))*(x-1)+1;
                                line4 := 6 x - 5

> plot({f(x),line1,line2,line3,line4},x=-3..6);

>
```

Observe that the expressions to be plotted are *enclosed* in braces `{ }`. Besides specifying the range on which `x` varies as done above, you can also specify and label the `y` range proceeding exactly in the same manner as when plotting a single function.

The most common errors in basic plotting are:

- Using a domain variable that already has a specific value.
- Not specifying a domain variable or specifying the wrong domain variable.

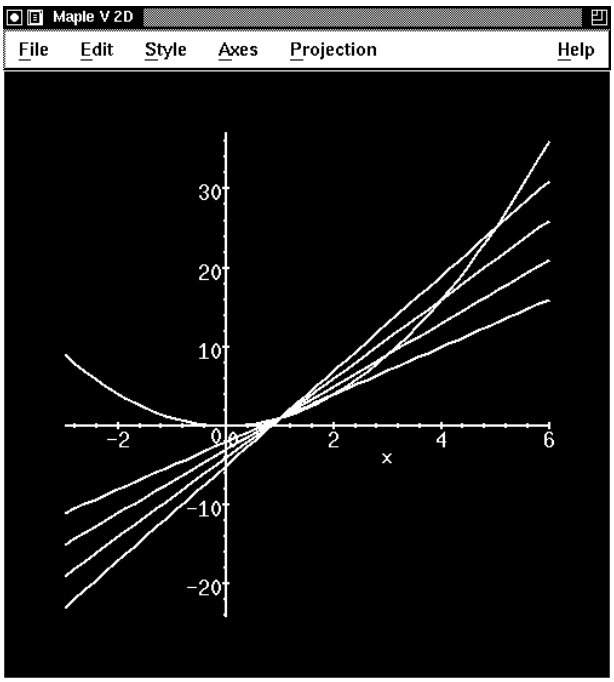

Figure 1.8.4: Window created by Maple with the plot of several functions on the same axes

1.8.2 Fancier plotting

The `plot` command is very powerful, and you should look for details about it using the online help facility:

```
> ?plot;
```

To make you aware of some of its features, we discuss them here:

- It is possible to have Maple *plot points*. This can be done in two ways depending upon how the points are generated. If you have a list of specific points, you can assign them to a name and then plot them as follows:

  ```
  > points:=[1,2],[1.5,1],[2,-1],[2.5,0.5],[3,1],[3.5,0.6],[4,0.2]:

  > plot([points],style=POINT);

  >
  ```

  If `style=POINT` is replaced by `style=LINE`, the dots will be connected by segments of lines.

On the other hand, if the points result from evaluating an expression at several values of `x`, you can use `plot` in its usual form, but specifying `style=POINT`:

```
> plot(x^2,x=-2..2,style=POINT);

>
```

Approximately 50 points will be plotted this way. You can insist that more points be plotted by using the *numpoints* option:

```
> plot(x^2,x=-2..2,style=POINT,numpoints=150);

>
```

- Maple's fanciest plotting functions are contained in a separate library called `plots`. For basic plotting, there are two commands from this library that are especially useful. They are `textplot` and `display`. To load these commands into the computer memory, execute the statement

```
> with(plots,textplot,display);
                              [display,textplot]
>
```

The `display` command is useful to combine different kinds of plots into one picture. You can merge standard plots of expressions, plots of points, plots of text (what `textplot` is for, useful for labelling things), and animations. In the next example, we combine the point plot of the variable `points` defined above with a plot of `sin(3*x)`. We first define them as different plots and then display them together:

```
> plot1:=plot([points],style=POINT):

> plot2:=plot(sin(3*x),x=0..4):

> display({plot1,plot2},view=[0..4,-1..2]);

>
```

Notice that `view` controls the horizontal and vertical ranges to be displayed.

You can attach labels to your plots by combining `textplot` with `display`. The basic syntax of `textplot` is to use it with an argument of the form `[a,b,'name']` which places the word name inside the quotes on the plot so that it is centered at the point (a, b). For example:

```
> y:=(1+x^2)*exp(-x^2/2): d:=diff(y,x):

> F:=plot({y,d},x=-3..3):

> G:=textplot({[1,1,'function'],[0.75,0.45,'derivative']}):

> display({F,G});

>
```

The result in this case is displayed in Fig. 5.

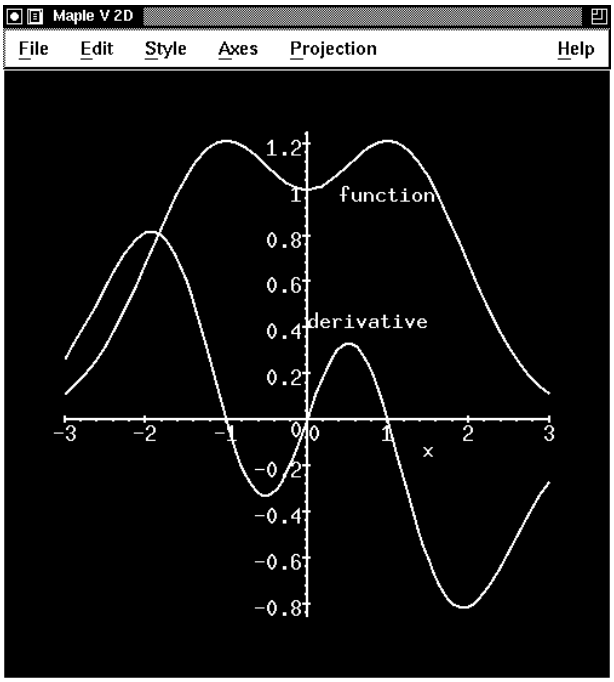

Figure 1.8.5: A plot generated combining textplot and display together

We end with a word about plotting functions (as opposed to expressions), and a situation in which it is a good idea to do so. Sometimes you will have a relationship that you want to plot in the form of a function rather than an expression:

```
> f:=x->x^3*exp(-x):
```

In this situation, you can plot `f(x)` as explained above. Alternatively, you may use:

```
> plot(f,0..3);

>
```

The two ways of plotting a function should not be confused. Neither of the following two statements will work correctly:

```
> plot(f(x),0..3);
Warning in iris-plot: empty plot

> plot(f,x=0..3);

>
```

If you have a function defined using an *if-then* clause, you must use the function plotting command:

```
> f:=x-> if x<3 then x+1 else -x^2+13 fi:

> plot(f(x),x=0..5);
Error, (in f) cannot evaluate boolean
```

The error resulted because Maple attempts to understand the function before it has a value for `x`. In this case, the command

```
> plot(f,0..5);

>
```

will work correctly as indicated in Fig. 6:
You can see from the graph that the function `f` is probably continuous but not differentiable at `x=3`.

1.8.3 Printing a graph

In the `File` menu at the top of any Maple window containing the plot of a curve (look at any of the previously given Figures), you can find a `Print` option which itself has several alternatives. You can save the figure into a postscript file by responding appropriately to the questions you are asked. You can give any name to the file where you want to save the figure, say `figure.ps`. Once you complete this processes, you will have a file with this name in the directory from which you launched Maple. You can print that file with the `lpr` command. However, you have to do so from any other window, not the Maple one.

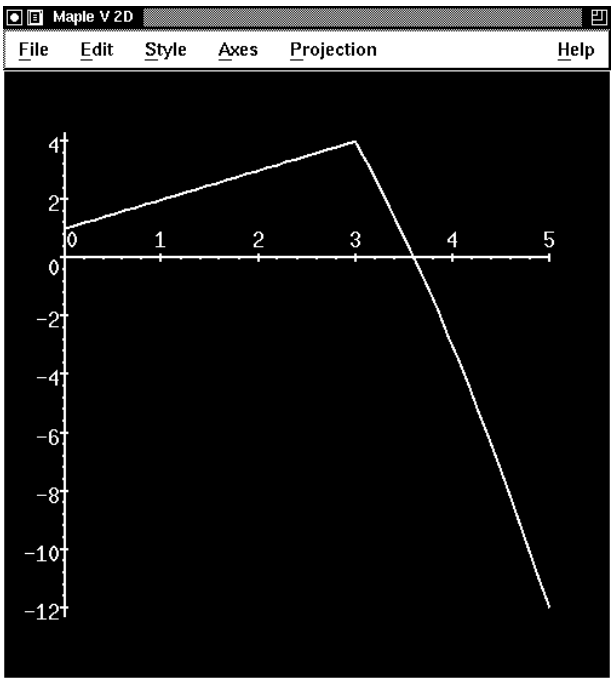

Figure 1.8.6: Plot of $f(x) = x + 1$ if $x < 3$ and $f(x) = -x^2 + 13$ if $x \geq 3$, displayed on the interval $[0, 5]$

1.9 Working with and from the Worksheet

Maple accepts your commands when the Maple prompt `>` shows on it. The session is in `Input mode`. Anything you type in while in this mode is understood as a Maple command and Maple will try to execute it. If the prompt is not present, the session is in `Text mode`. You can type anything you want and Maple will interpret it as text to be recorded but not executed. You can move from the `Input mode` to the `Text mode` by positioning the cursor on the `Input` box and holding down its left button. Such an action will display a menu from which you select the `Text` item. Similarly, you can move back from `Text` mode to `Input` mode by positioning the cursor on the `Edit` menu, holding down the left button and selecting the `Insert Prompt` from the submenu.

To make your worksheets easy to read, in particular, when you have to turn them in as homework, you should mix some amount of text with your Maple commands explaining to the reader what is being done. This will be very useful for you when you save your Maple session and resume work from the saved worksheet at a later date. By that time, you may have forgotten all that you were doing and the inserted text will make easier for you to remember it.

1.9.1 Saving a resuming work from a worksheet

You may save your Maple session to a file so that you can use it again at a later date: with the cursor on the `File` menu of the Maple window, press the left button of the mouse and select the the entry `Save As...`. A small window will appear where you can choose the directory where you want to save the worksheet. In the `Selection` field of this window you can type in a name under which you want to save it.

You may resume work on a previously saved worksheet: use the left button to click on File and select Open from the menu. A Load Session window will appear from where you can choose the file you want to use. Click on it.

When working of a set of Maple problems, it is a good idea to start a new session and worksheet for each problem. This avoids the problem of remembering which variables you have used before and saves you from loosing a whole assignment if something goes wrong with the computer. Try to save your worksheets often, in particular after doing something that you do not want to do again.

1.9.2 Quitting a Maple session and logging out

To quit a Maple session place the cursor on the `File` menu, and while holding down the left button of the mouse select the `Exit` option. You may or may not want to save the worksheet as indicated previously. Answer the question accordingly. To log out of the system, position the cursor in the background and select the `Exit` option. This will terminate your login session.

1.10 Exercises

1. Evaluate $\pi^{\sqrt{2}}$ to 30 decimal places.

2. Input the following as a Maple function:

$$f(x) = \frac{\sqrt{x}(-116x^3 + 8x^4 + 558x^2 - 891x)}{2x^5 - 29x^4 + 140x^3 - 225x^2}\,.$$

   a) Write this in a simpler form (that is, factor and reduce it).

   b) Draw the graph of the function for x between 0 and 1). Adjust the vertical range so that some detail can be seen.

   c) Compute the area of the part of the curve that lies above the x-axis and between $x = 0$ and $x = 5$ (that is, integrate the function over the appropriate range of x values). Give your answer both in actual form and as a decimal approximation to about 10 places.

   d) What is the value of the integral if you remove the factor of $\sqrt{x}$ from the numerator?

e) Use the derivative of $f(x)$ to determine for what real value x the function has a local maximum. Use an approximation to about 8 decimal places.

3. Use the commands `seq` and `ithprime` to generate a list of the first 20 primes. Compute the sum of these 20 primes, and give its integer factors.

4. Find the solutions of the system of equations $\{x^2 - y^2 = 4, x - 2y = 2\}$.

5. Draw a graph showing both $\cos x$ and its fifth Taylor polynomial (that is, $1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4$) for x between -4 and 4. How many terms do you seem to need to get *good agreement* in this range.

   Hint: use a variation of the command `convert(taylor(cos(x),x,5),polynom)` to make this work.

   Think of a suitable way to demonstrate that the approximation you have taken is *good*.