

MAT 342, Homework 6 due 10/9

Questions from the textbook Sec. 49 p. 147: question **3**; Sec. 53 p. 159 questions **1bcf**, **2abc**.

More questions:

1. Find antiderivatives and compute the following integrals over the given paths. Please *do not* work with the parameterizations.

$$(a) \int_C \cos\left(\frac{z}{2}\right) dz, \quad \text{where } C = \{te^{it}, 2\pi \leq t \leq 4\pi\}.$$

$$(b) \int_C \frac{z}{2z^2 - 1} dz, \quad \text{where } C \text{ is formed by the segments } \{x = 0, 0 \leq y \leq 1\}, \{0 \leq x \leq 2, y = 1\}, \{x = 2, 0 \leq y \leq 1\}.$$

2. Let C_R be the circle $|z| = R$, with the counterclockwise direction. Show that for $R > 1$,

$$\left| \int_{C_R} \frac{\operatorname{Log} z}{z^2} dz \right| \leq 2\pi \left(\frac{\pi + \ln R}{R} \right).$$

Furthermore, show that

$$\lim_{R \rightarrow \infty} \int_{C_R} \frac{\operatorname{Log} z}{z^2} dz = 0.$$

Optional: read the proof of the Cauchy–Goursat theorem without the extra assumptions, section 51. Do questions **4**, **5**, **7** in section 53 pp. 159–161. (The questions are unrelated to the reading, so you can do whichever part you like.)