Lecture 7

Sets

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A set and its elements

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In any intellectual activity,
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is organized and regulated by the **naive set theory**.

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The **first words** in this language are **set** and **element**.

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A **set** is a collection of objects which are called **elements**.

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The **first words** in this language are **set** and **element**.

A **set** is a collection of objects which are called **elements**.
A set **consists** of (and is **defined** by) its elements.

Notations and synonims

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Notation: $x \in S$ "x is an **element** of a set S"

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Standard number sets

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Definition. An **empty set**

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empty box $\neq$ a box containing an empty box.

Subsets

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Example. $A = \{1, 2, 3\}$.

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correct: $1 \in A$ $\{1\} \in A$

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if any element of A is an element of B.

Notation: $A \subset B$, or $A \subseteq B$, or $B \supset A$, or $B \supseteq A$.

The signs "$\subset$","$\subseteq$", "$\supset$" and "$\supseteq$" are called **inclusion** symbols.
Commonly $\subset$ and $\subseteq$ are used in the same sense.

By definition, $A \subset B \iff \forall x\, (x \in A \implies x \in B)$.

Warning: distinguish the signs "$\in$" and "$\subset$"

Example. $A = \{1, 2, 3\}$.

correct: $1 \in A$ wrong: $\{1\} \in A$

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Example. $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$.

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Intersection and Union

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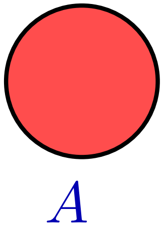

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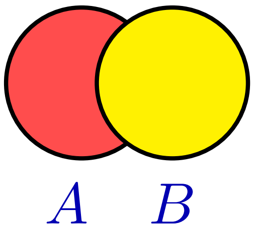

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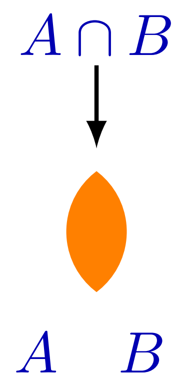

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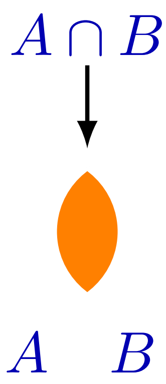

Venn diagram

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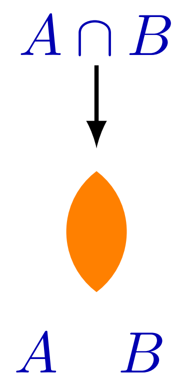

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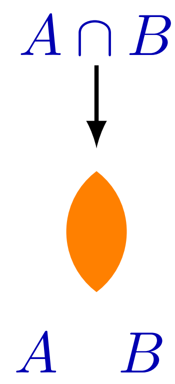

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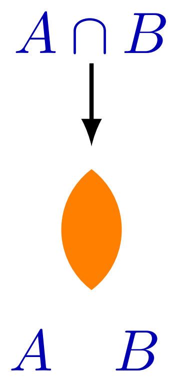

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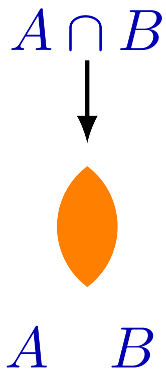

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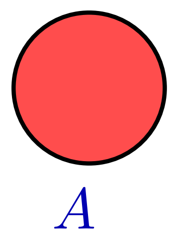

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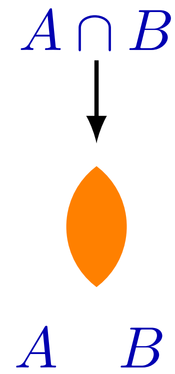

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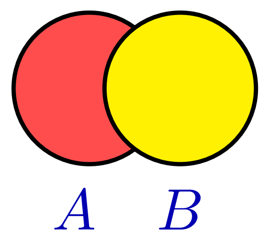

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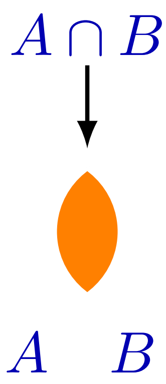

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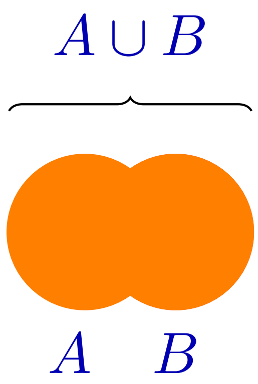

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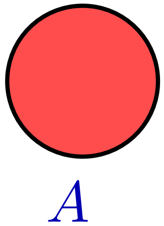

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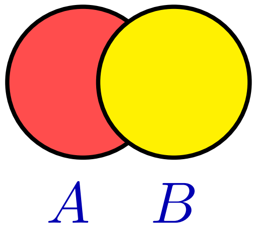

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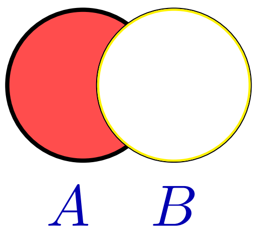

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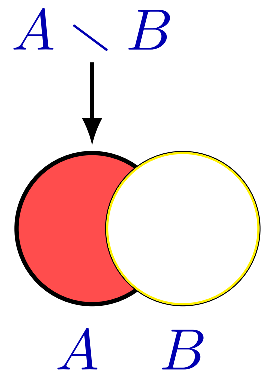

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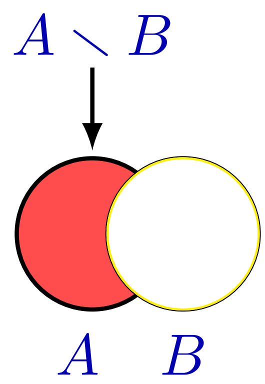

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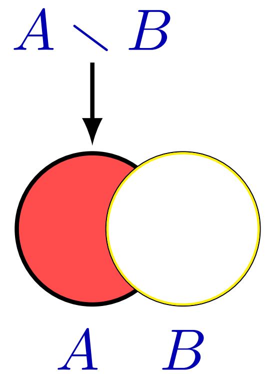

Complement

A^C

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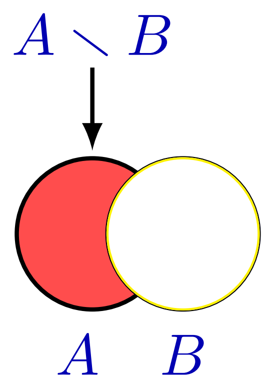

Complement

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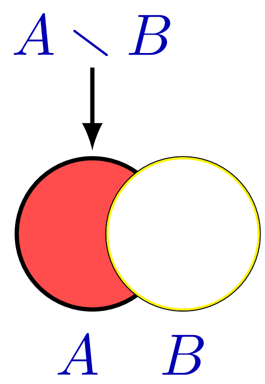

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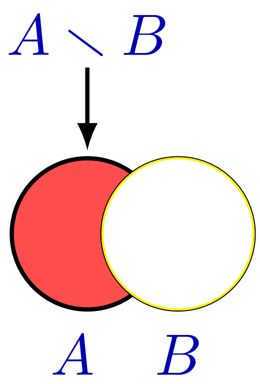

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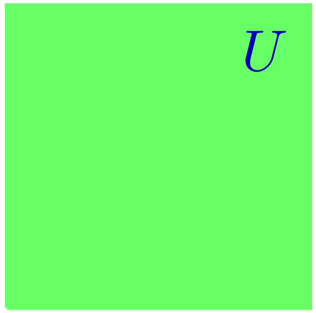

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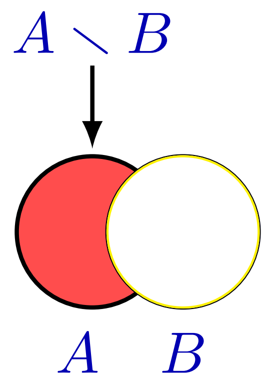

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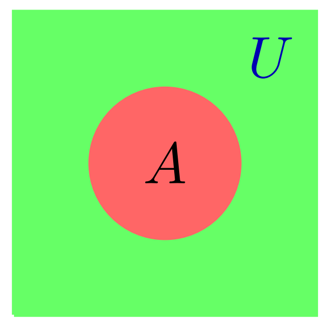

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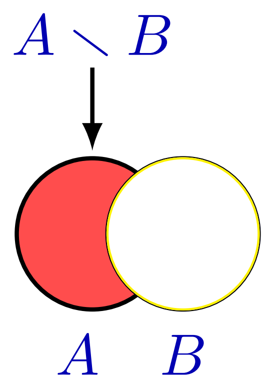

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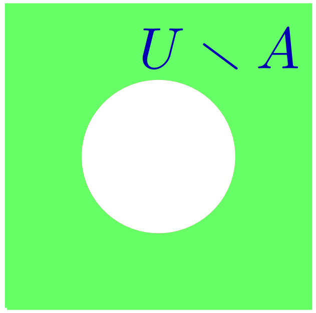

Simplest set-theoretical identities

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The subset of a set A consisting of the elements x that satisfy a condition $P(x)$ is denoted by $\{x \in A \mid P(x)\}$.

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Vice versa, every subset $B \subset A$ gives rise to a predicate $x \in B$.

Logic vs. set theory

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Proving set-theoretic identities: De Morgan's law

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Example 1.

Proving set-theoretic identities: De Morgan's law

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Therefore, $(A \cap B)^c \subset A^c \cup B^c \quad (*)$.

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Prove now that $A^c \cup B^c \subset (A \cap B)^c$.

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$x \in A^c \cup B^c$

Proving set-theoretic identities: De Morgan's law

Example 1. Prove **De Morgan's** law: $(A \cap B)^c = A^c \cup B^c$

Proof. Let us prove first that $(A \cap B)^c \subset A^c \cup B^c$.

Let $x \in (A \cap B)^c$ Then $x \notin A \cap B \implies \neg(x \in A \wedge x \in B)$

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Combining $(*)$ and $(**)$, we get $(A \cap B)^c = A^c \cup B^c$. □

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How to prove set-theoretic identities

Example 2.

How to prove set-theoretic identities

Example 2. Prove that $A \setminus B = A \cap B^c$

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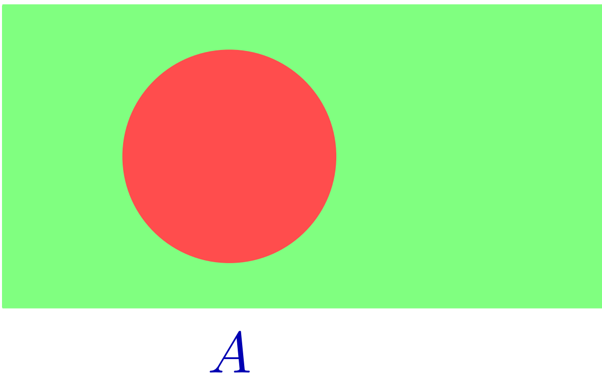

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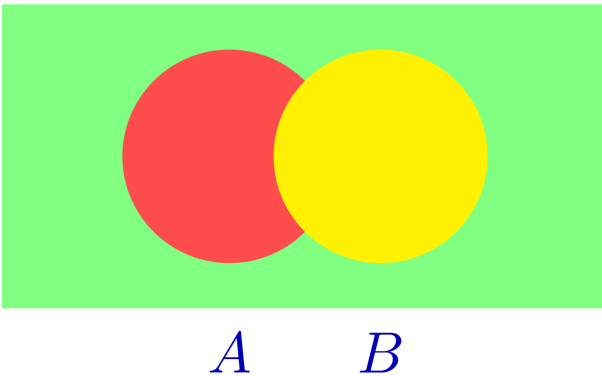

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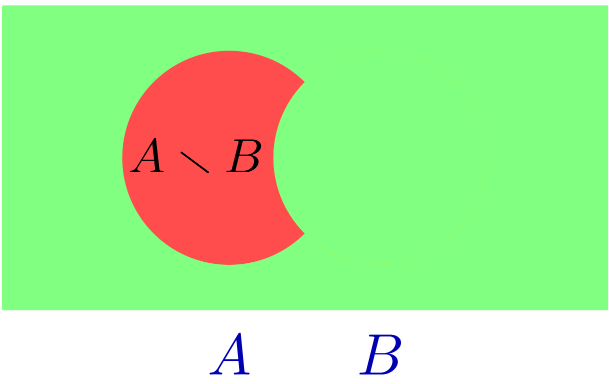

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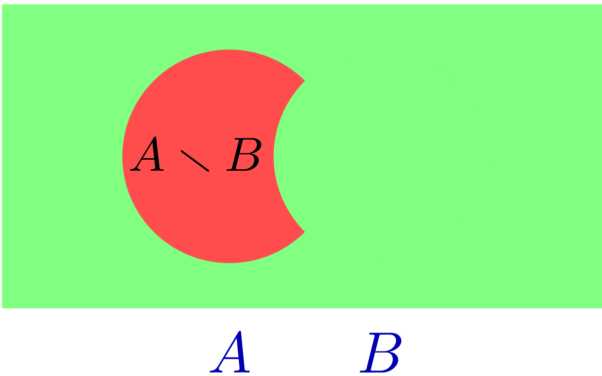

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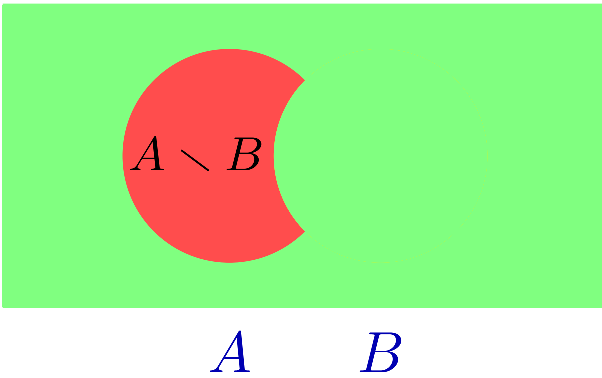

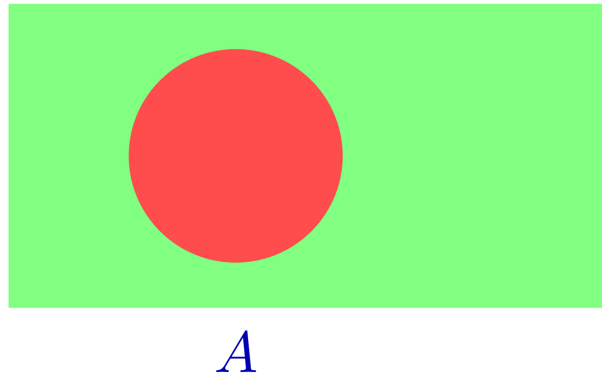

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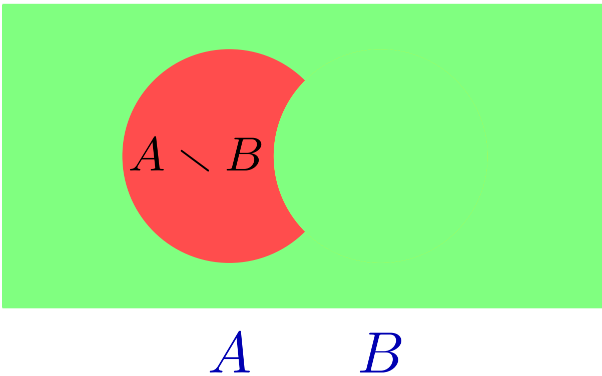

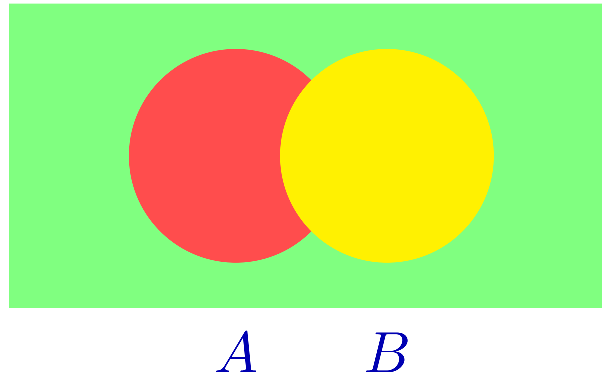

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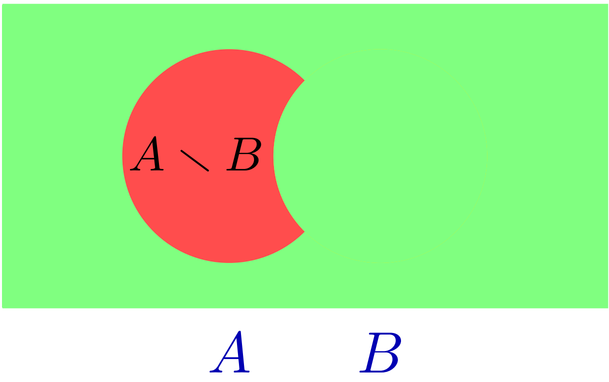

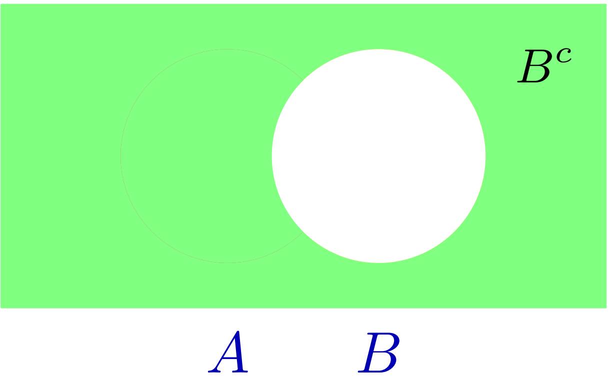

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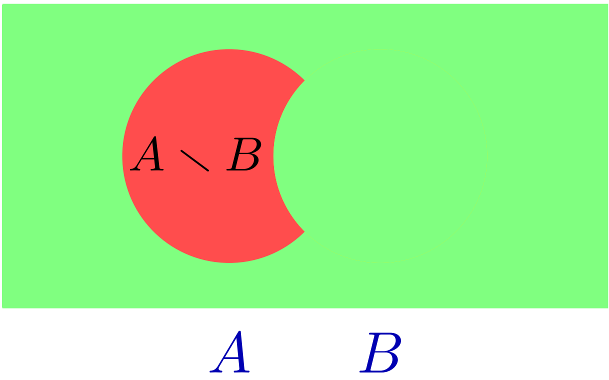

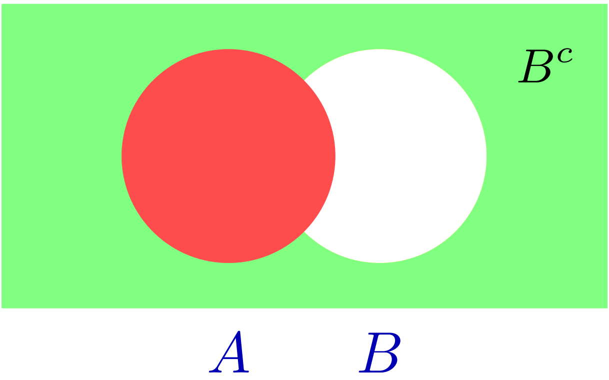

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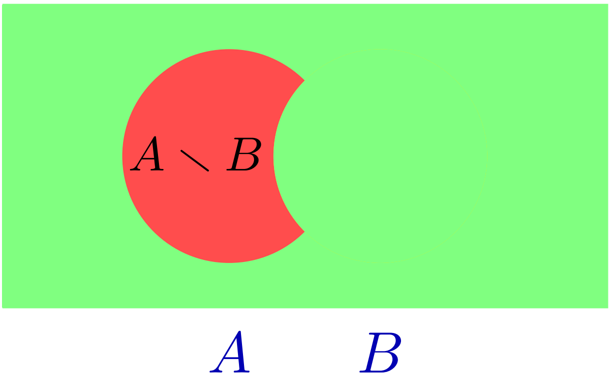

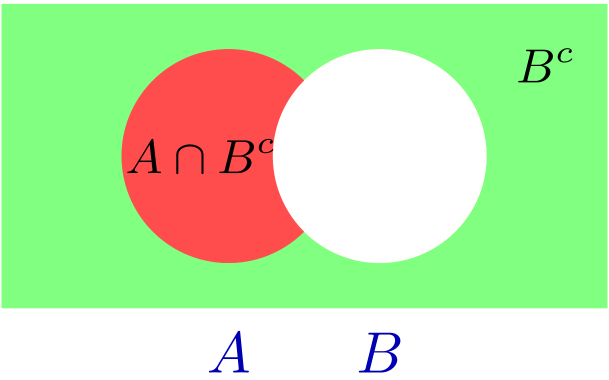

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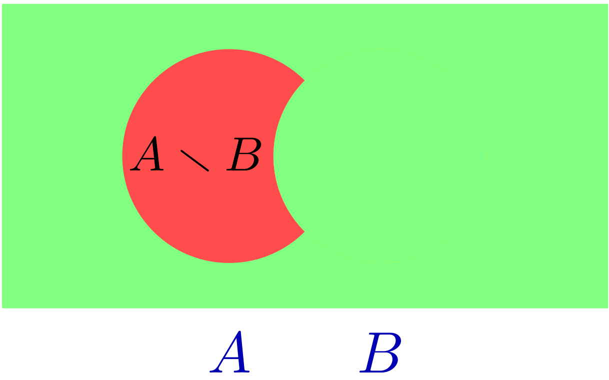

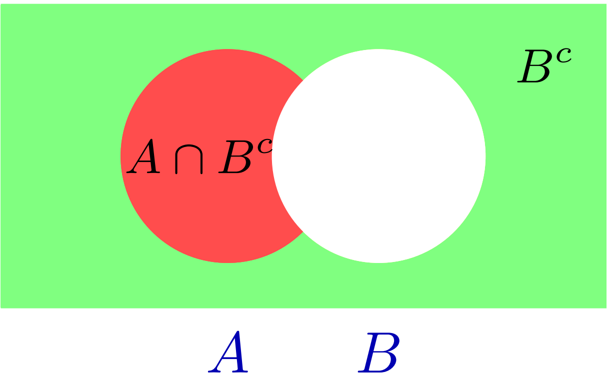

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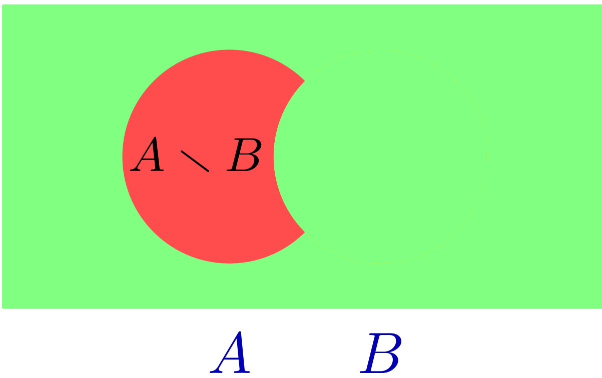

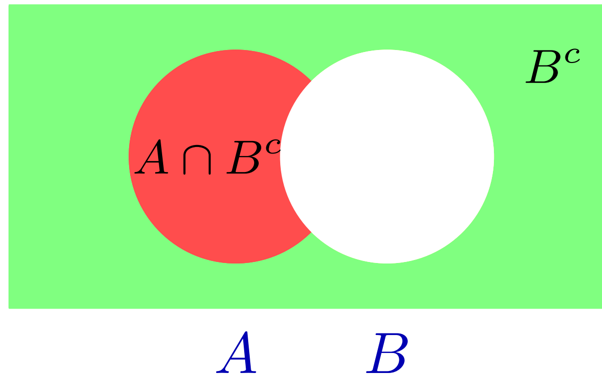

Proof. Alternative 1

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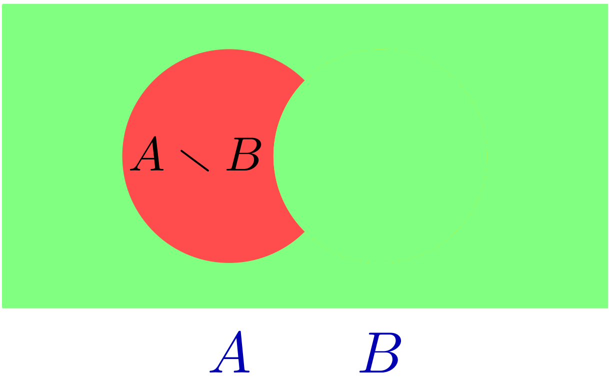

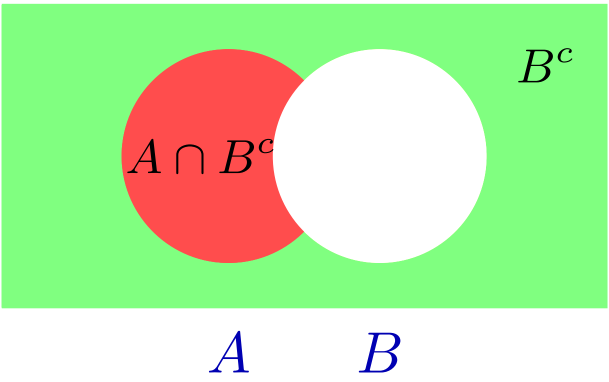

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How to prove set-theoretic identities

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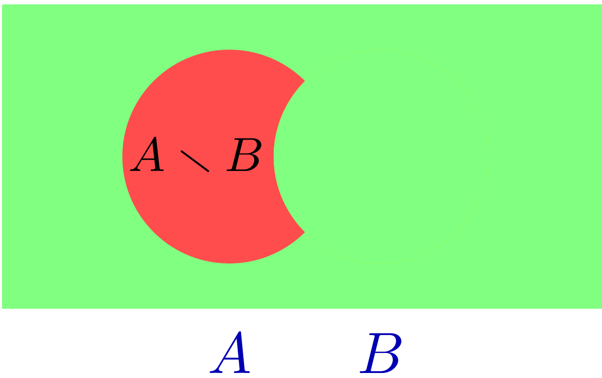

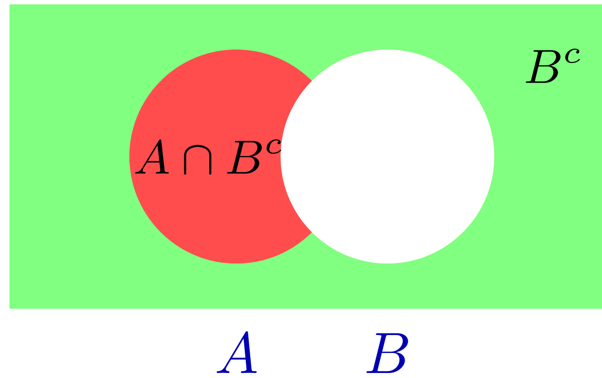

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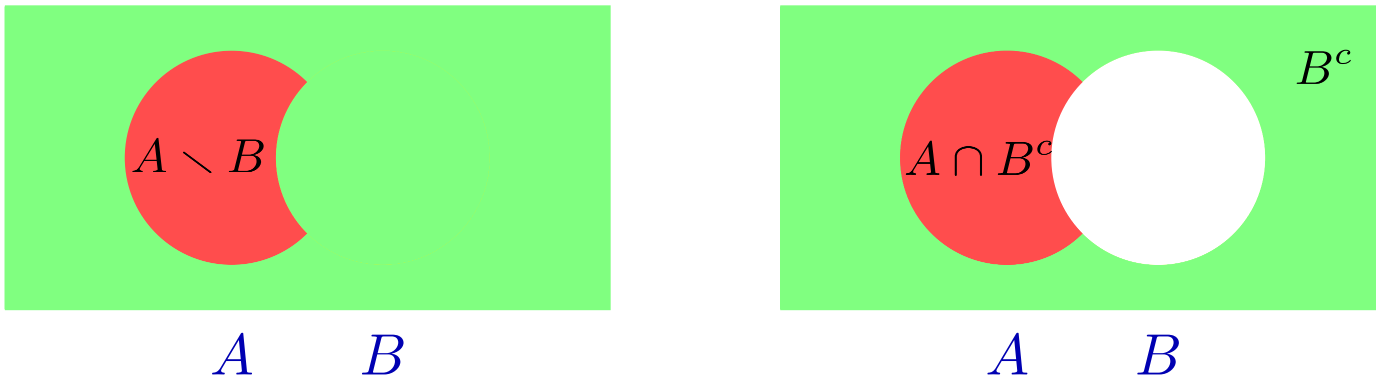

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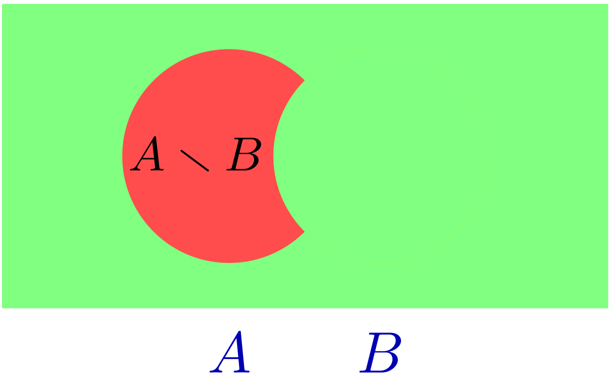

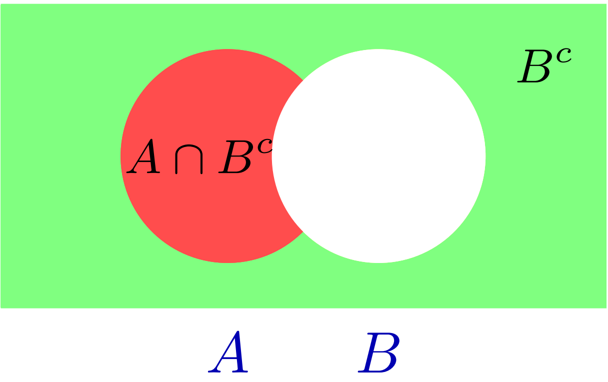

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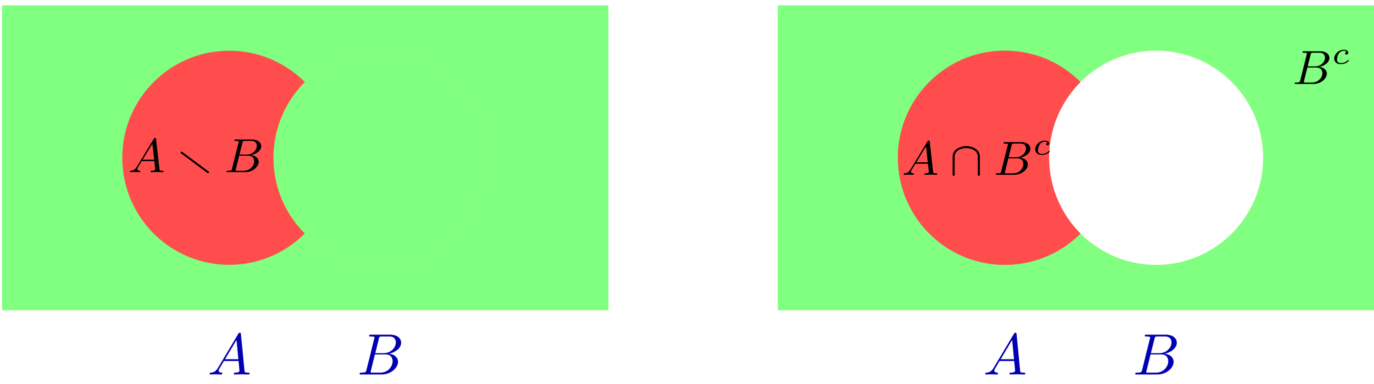

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How to prove set-theoretic identities

Alternative 2

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Alternative 2 (two sets are equal iff each of them is a subset of the other)

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To prove $A \smallsetminus B = A \cap B^c$, we prove that

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To prove $A \setminus B = A \cap B^c$, we prove that

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How to prove set-theoretic identities

Alternative 2 (two sets are equal iff each of them is a subset of the other)

To prove $A \setminus B = A \cap B^c$, we prove that

$$A \setminus B \subset A \cap B^c \quad \text{and}$$
$$A \setminus B \supset A \cap B^c$$

Indeed,

$$A \setminus B \subset A$$
$$A \setminus B \subset U \setminus B = B^c$$

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We have got that $A \setminus B \subset A \cap B^c$ and $A \setminus B \supseteq A \cap B^c$.

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Therefore, $A \setminus B = A \cap B^c$. □

How to prove set-theoretic identities

Alternative 3

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Alternative 3 (by truth table)

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	$x \in A$	$x \in B$	$x \notin B$	$\underbrace{x \in A \wedge x \notin B}_{x \in A \smallsetminus B}$	$\underbrace{x \in A \wedge x \in B^c}_{x \in A \cap B^c}$
1	T	T	F	F	F
2	T	F	T	T	T
3	F	T	F	F	F
4	F	F	T	F	F

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Since the last two columns of the truth table are identical,

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Since the last two columns of the truth table are identical, $A \setminus B = A \cap B^c$.

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Since the last two columns of the truth table are identical, $A \setminus B = A \cap B^c$.

Remark. The universe can be presented as a **disjoint union**

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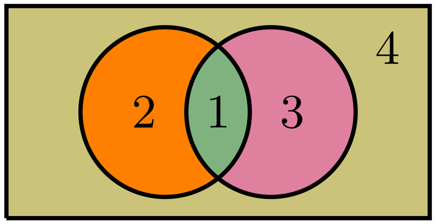
Since the last two columns of the truth table are identical, $A \setminus B = A \cap B^c$.

Remark. The universe can be presented as a **disjoint union**

$$U = (A \cap B) \cup (A \setminus B) \cup (B \setminus A) \cup (A \cup B)^c$$



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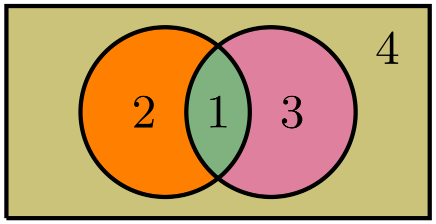

What does this formula remind you?

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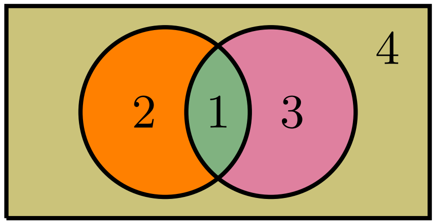

What does this formula remind you? Is it related to disjunctive normal form?

How to prove set-theoretic identities

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Example 3.

How to prove set-theoretic identities

Example 3. Prove that $A \subset B \iff A \setminus B = \varnothing$

How to prove set-theoretic identities

Example 3. Prove that $A \subset B \iff A \setminus B = \varnothing$ for any sets A, B.

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Proof.

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Example 3. Prove that $A \subset B \iff A \setminus B = \varnothing$ for any sets A, B.

Proof. Let $A \subset B$.

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Proof. Let $A \subset B$. Then
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Example 3. Prove that $A \subset B \iff A \setminus B = \varnothing$ for any sets A, B.

Proof. Let $A \subset B$. Then
$\forall x \in A \quad x \in A \implies x \in B \implies x \notin B^c$.
So any x in A doesn't belong to B^c.

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Proof. Let $A \subset B$. Then
$\forall x \in A \quad x \in A \implies x \in B \implies x \notin B^c$.
So any x in A doesn't belong to B^c. Therefore,

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So any x in A doesn't belong to B^c. Therefore, $A \cap B^c = \varnothing$.

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We have proven that $A \subset B \implies A \setminus B = \varnothing \quad (*)$

Prove now the opposite implication.

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How to prove set-theoretic identities

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Combining $(*)$ and $(**)$, we get $A \subset B \iff A \setminus B = \varnothing$. □

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