Lecture 2

Quantifiers

Introduction to quantifiers

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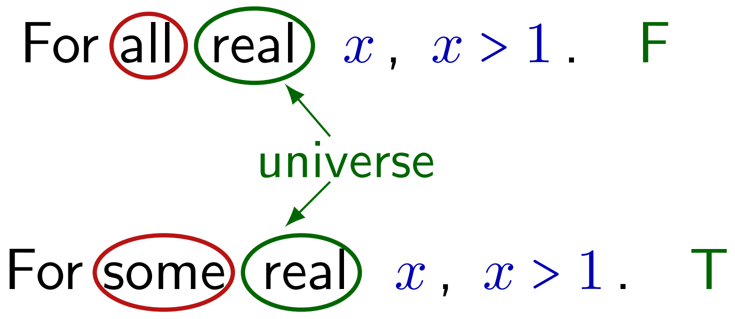

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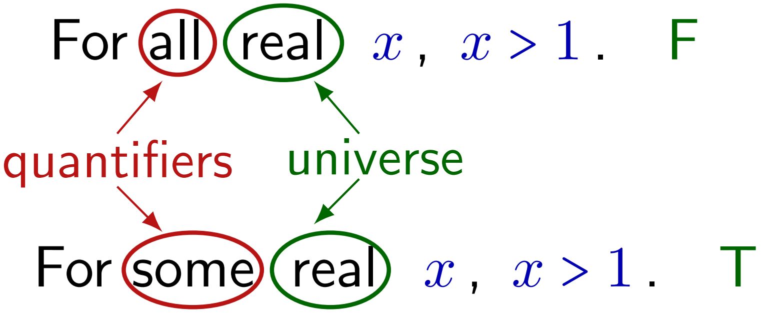

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A variable which is **not** bound by any quantifier is called **free**.

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- For **each** x, $P(x)$
- For **every** x, $P(x)$
- For **arbitrary** x, $P(x)$

The quantified sentence $\exists x\, P(x)$ has the following wordings:

- For **some** x, $P(x)$
- There **exists** x such that $P(x)$

Unique existential quantifier $\exists!$

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There exists a unique integer x such that $1 < x < 3$.

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The unique existential quantifier can be expressed in terms of the universal and existential quantifiers:

$$\exists!\, x\, P(x) \iff (\exists\, x\, P(x)) \wedge (\forall x_1 \forall x_2\, P(x_1) \wedge P(x_2) \implies x_1 = x_2)$$

From symbols to words

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Example. Let B be the set of all birds, and $F(x)$ be the predicate " x can fly."

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From symbols to words

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Example. Let the universe (where the variable take values) be the set of people.

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$\forall\, x\ \ (W(x) \implies S(x))$ Each person who likes whisky is a Scot.

$\exists\, x\ \ (S(x) \wedge \neg\, W(x))$ Some Scots don't like whisky.

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$\forall\, x\ \ (\neg\, W(x) \implies \neg\, S(x))$	Anyone who doesn't like whisky can't be a Scot.

From words to symbols

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We need to introduce a **variable** for an integer.

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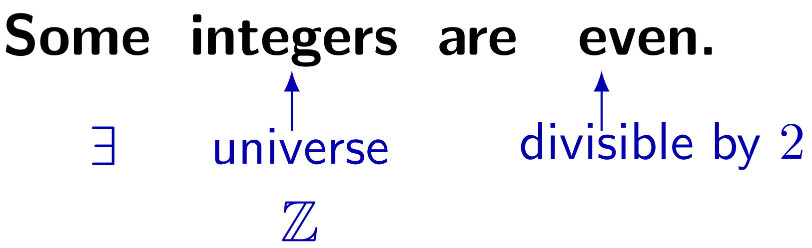

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Example 1. Write the following sentence in a **symbolic** form:

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Every rational number is a real number.

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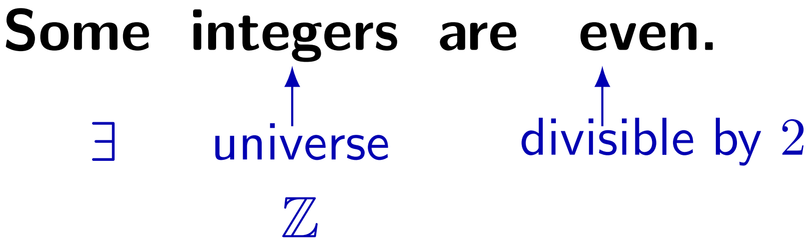

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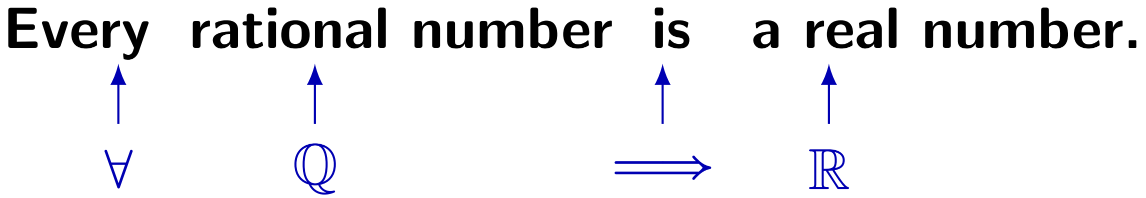

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Do we understand quantified sentences?

Determine the **truth value** for each of the following statements:

$\exists\, n \in \mathbb{Z}$	$n^2 = 9$	T	Indeed, there exists such n, namely $n = 3$.
$\exists!\, n \in \mathbb{Z}$	$n^2 = 9$	F	Such n is **not** unique: $n = 3$ and $n = -3$ will work.
$\forall\, n \in \mathbb{Z}$	$n^2 = 9$	F	**Not all** n posses this property. For example, $n = 1$ doesn't work.
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Therefore, the proof is incorrect.

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Example. For the sentence "**All integers are even**",

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$$\neg(\exists\, x\, P(x)) \iff \forall\, x\ \neg P(x)$$

Example. For the sentence “**All integers are even**”, construct a denial (both in words and symbols).

Solution. We start with writing the sentence in a symbolic form.

$$\forall n(n \in \mathbb{Z} \implies 2 \mid n)$$

Let us construct a useful denial of this sentence.

☞ remember that that implication $P \implies Q$ is negated as $P \wedge \neg Q$:

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$\exists n\,(n \in \mathbb{Z} \wedge 2 \nmid n)$ says “**Some integers are not even.**”

Passing negation through quantifies

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$$\exists\, x\ \forall\, y\ \forall\, z\ \exists\, w\ \neg\,(P(x,z) \implies Q(y,w)) \iff$$

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Quantified sentences with two quantifiers

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Only quantifiers of the same type **commute**.

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Only quantifiers of the same type **commute**.
The existential quantifier and the universal quantifier do **not** commute.

$$\begin{tikzcd}
& \exists x \, \forall y \, P(x,y) \arrow[r, Rightarrow] & \forall y \, \exists x \, P(x,y) \arrow[dr, Rightarrow] & \\
\forall x \, \forall y \, P(x,y) \arrow[ur, Rightarrow] \arrow[dr, Rightarrow] & & & \exists x \, \exists y \, P(x,y) \\
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\end{tikzcd}$$

Different quantifiers don't commute

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Determine the truth value of the following sentences:

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Example 2.

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Example 2. Let $x, y \in \mathbb{R}$. Determine the truth value of the following sentences:

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Example 2. Let $x, y \in \mathbb{R}$. Determine the truth value of the following sentences:

$\forall x \;\; \exists y \;\; x = y$

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Different quantifiers don't commute

Example 1. Let C be the set of all children and F be the set of all fathers. Let $P(x,y)$ be the predicate "y **is the father of** x**.**"

Determine the truth value of the following sentences:

$\forall x \in C \;\; \exists y \in F \;\; P(x,y)$ Each child has a father. T

$\exists y \in F \;\; \forall x \in C \;\; P(x,y)$ There exists a person who is the father of all children. F

Example 2. Let $x, y \in \mathbb{R}$. Determine the truth value of the following sentences:

$\forall x \;\; \exists y \;\; x = y$ T

$\exists y \;\; \forall x \;\; x = y$ F

Example 3. True of false?

$\forall a \in \mathbb{R} \;\; \exists x \in \mathbb{C} \;\; x^2 = a$ T

$\exists x \in \mathbb{C} \;\; \forall a \in \mathbb{R} \;\; x^2 = a$ F

All quantified sentences

All quantified sentences

Problem. Construct all possible quantified sentences using

All quantified sentences

Problem. Construct all possible quantified sentences using real variables x, y ,

All quantified sentences

Problem. Construct all possible quantified sentences using
real variables x, y ,
the universal and existential quantifiers,

All quantified sentences

Problem. Construct all possible quantified sentences using
real variables x, y,
the universal and existential quantifiers, and
the predicate $x + y = 0$.

All quantified sentences

Problem. Construct all possible quantified sentences using
real variables x, y,
the universal and existential quantifiers, and
the predicate $x + y = 0$.
Determine the truth value of the sentences.

All quantified sentences

Problem. Construct all possible quantified sentences using
real variables x, y ,
the universal and existential quantifiers, and
the predicate $x + y = 0$.
Determine the truth value of the sentences. Justify your answer.

All quantified sentences

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Solution.

All quantified sentences

Problem. Construct all possible quantified sentences using
real variables x, y,
the universal and existential quantifiers, and
the predicate $x + y = 0$.

Determine the truth value of the sentences. Justify your answer.

Solution. There are $\underline{\text{eight}}$ quantified sentences:

All quantified sentences

Problem. Construct all possible quantified sentences using
real variables x, y,
the universal and existential quantifiers, and
the predicate $x + y = 0$.

Determine the truth value of the sentences. Justify your answer.

Solution. There are eight quantified sentences:

1) $\exists\, x \;\; \exists\, y \quad x + y = 0$

2) $\exists\, x \;\; \forall\, y \quad x + y = 0$

3) $\forall\, x \;\; \exists\, y \quad x + y = 0$

4) $\forall\, x \;\; \forall\, y \quad x + y = 0$

5) $\exists\, y \;\; \exists\, x \quad x + y = 0$

6) $\exists\, y \;\; \forall\, x \quad x + y = 0$

7) $\forall\, y \;\; \exists\, x \quad x + y = 0$

8) $\forall\, y \;\; \forall\, x \quad x + y = 0$

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3) $\forall x \ \exists y \quad x + y = 0$
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8) $\forall y \ \forall x \quad x + y = 0$

Let us determine the truth value of each quantified sentence.

All quantified sentences

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2) $\exists x \ \forall y \quad x + y = 0$	**6)** $\exists y \ \forall x \quad x + y = 0$
3) $\forall x \ \exists y \quad x + y = 0$	**7)** $\forall y \ \exists x \quad x + y = 0$
4) $\forall x \ \forall y \quad x + y = 0$	**8)** $\forall y \ \forall x \quad x + y = 0$

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1) $\exists x \ \exists y \quad x + y = 0$ T

All quantified sentences

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8) $\forall y \ \forall x \quad x + y = 0$

Let us determine the truth value of each quantified sentence.

1) $\exists x \ \exists y \quad x + y = 0$ T Why?

All quantified sentences

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real variables x, y,
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1) $\exists\, x \;\; \exists\, y \quad x + y = 0$ T Why?

Indeed, there exists x,

All quantified sentences

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1) $\exists x \ \exists y \ \ x + y = 0$ T Why?

Indeed, there exists x, for example, $x = 2$,

All quantified sentences

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Let us determine the truth value of each quantified sentence.

1) $\exists\, x \;\; \exists\, y \quad x + y = 0$ T Why?

Indeed, there exists x, for example, $x = 2$, and there exists y, namely $y = -2$,

All quantified sentences

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real variables x, y,
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Let us determine the truth value of each quantified sentence.

1) $\exists x \ \exists y \quad x + y = 0$ T Why?

Indeed, there exists x, for example, $x = 2$, and there exists y, namely $y = -2$, such that $\underbrace{x}_{2} + \underbrace{y}_{-2} = 0$.

All quantified sentences

All quantified sentences

2) $\exists\, x \;\; \forall\, y \;\;\; x + y = 0$

All quantified sentences

2) $\exists\, x \;\; \forall\, y \;\;\; x + y = 0$ F

All quantified sentences

2) $\exists\, x \quad \forall\, y \quad x + y = 0$ F Why?

All quantified sentences

2) $\exists\, x \;\; \forall\, y \;\;\; x + y = 0$ F Why?

The statement says

All quantified sentences

2) $\exists x \ \forall y \quad x + y = 0$ F Why?

The statement says that there exists **some** special number x,

All quantified sentences

2) $\exists\, x \;\; \forall\, y \quad x + y = 0$ F Why?

The statement says that there exists **some** special number x,
to which we can add **any** number y

All quantified sentences

2) $\exists x \ \forall y \ \ x + y = 0$ F Why?

The statement says that there exists **some** special number x,
to which we can add **any** number y and get 0 as the result.

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The statement says that there exists **some** special number x, to which we can add **any** number y and get 0 as the result.

This can't be true,

All quantified sentences

2) $\exists x \;\; \forall y \;\; x + y = 0$ F Why?

The statement says that there exists **some** special number x, to which we can add **any** number y and get 0 as the result.

This can't be true, such special number doesn't exist.

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The statement says that there exists **some** special number x,
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The justification above requires many words.

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All quantified sentences

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$\forall\, x \;\; \exists\, y \;\;\; x + y \neq 0.$

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$\forall\, x \;\; \exists\, y \quad x + y \neq 0.$ This is true, since for any x there exists y,

All quantified sentences

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The statement says that there exists **some** special number x, to which we can add **any** number y and get 0 as the result. This can't be true, such special number doesn't exist.

The justification above requires many words. Let us see how to reason more efficiently. Consider the **denial** of the statement:

$\forall x \ \exists y \quad x + y \neq 0.$ This is true, since for any x there exists y, namely, $y = 1 - x$,

All quantified sentences

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$\forall\, x \;\; \exists\, y \quad x + y \neq 0.$ This is true, since for any x there exists y, namely, $y = 1 - x$, such that $x + \underbrace{y}_{1-x} \neq 0$.

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Therefore, the denial is true,

All quantified sentences

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Therefore, the denial is true, and the original statement is **false**.

All quantified sentences

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All quantified sentences

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The statement $\exists x \ \forall y \ \ x + y = 0$ **is true,**

All quantified sentences

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Therefore, the denial is true, and the original statement is **false**.

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The statement $\exists\, x \;\; \forall\, y \quad x + y = 0$ **is true,**
since there is x**,**

All quantified sentences

2) $\exists\, x \;\; \forall\, y \;\;\; x + y = 0$ F Why?

The statement says that there exists **some** special number x, to which we can add **any** number y and get 0 as the result.
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Therefore, the denial is true, and the original statement is **false**.

Warning. Here is a typical example of **incorrect** reasoning.

The statement $\exists\, x \;\; \forall\, y \;\;\; x + y = 0$ is true,
since there is x, namely $x = -y$,

All quantified sentences

2) $\exists x \ \forall y \ \ x + y = 0$ F Why?

The statement says that there exists **some** special number x, to which we can add **any** number y and get 0 as the result.

This can't be true, such special number doesn't exist.

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The statement $\exists x \ \forall y \ \ x + y = 0$ **is true,**
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All quantified sentences

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The statement $\exists\, x\ \ \forall\, y\ \ \ x + y = 0$ is true,
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Why this reasoning is incorrect?

All quantified sentences

2) $\exists x \ \forall y \ \ x + y = 0$ F Why?

The statement says that there exists **some** special number x, to which we can add **any** number y and get 0 as the result.
This can't be true, such special number doesn't exist.

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Therefore, the denial is true, and the original statement is **false**.

❦ **Warning.** Here is a typical example of **incorrect** reasoning.

The statement $\exists x \ \forall y \ \ x + y = 0$ **is true,**
since there is x**, namely** $x = -y$**, such that for all** y **we have** $\underbrace{x}_{-y} + y = 0$**.**

Why this reasoning is incorrect? The statement claims existence of some x that serves all y. So x should <u>not</u> depend on y.

All quantified sentences

3) $\forall\, x \;\; \exists\, y \quad x + y = 0$

All quantified sentences

3) $\forall x \;\; \exists y \quad x + y = 0$ T

All quantified sentences

3) $\forall x \ \exists y \quad x + y = 0$ T Why?

All quantified sentences

3) $\forall x \;\; \exists y \quad x + y = 0$ T Why?

No matter which x is chosen,

All quantified sentences

3) $\forall x \;\; \exists y \;\;\; x + y = 0$ T Why?

No matter which x is chosen, we may always find y ,

All quantified sentences

3) $\forall\, x\ \ \exists\, y\ \ \ x + y = 0$ T Why?

No matter which x is chosen, we may always find y, namely, $y = -x$,

All quantified sentences

3) $\forall x \ \exists y \quad x + y = 0$ T Why?

No matter which x is chosen, we may always find y, namely, $y = -x$,
such that $x + \underbrace{y}_{-x} = 0$

All quantified sentences

3) $\forall\, x \;\; \exists\, y \;\; x + y = 0$ T Why?

No matter which x is chosen, we may always find y, namely, $y = -x$, such that $x + \underbrace{y}_{-x} = 0$

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No matter which x is chosen, we may always find y, namely, $y = -x$,

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The statement $\forall\, x \;\; \exists\, y \;\; x + y = 0$ **is true.**

All quantified sentences

3) $\forall x \ \exists y \ \ x + y = 0$ T Why?

No matter which x is chosen, we may always find y, namely, $y = -x$,
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Warning. Here is a typical example of **incorrect** reasoning.

The statement $\forall x \ \exists y \ \ x + y = 0$ **is true.**

Indeed, for $x = 2$ **,**

All quantified sentences

3) $\forall x \;\; \exists y \quad x + y = 0$ T Why?

No matter which x is chosen, we may always find y, namely, $y = -x$,
such that $x + \underbrace{y}_{-x} = 0$

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The statement $\forall x \;\; \exists y \quad x + y = 0$ **is true.**

Indeed, for $x = 2$**, there exists** y**, namely** $y = -2$**,**

All quantified sentences

3) $\forall\, x \;\; \exists\, y \;\; x + y = 0$ T Why?

No matter which x is chosen, we may always find y, namely, $y = -x$, such that $x + \underbrace{y}_{-x} = 0$

❦ **Warning.** Here is a typical example of **incorrect** reasoning.

The statement $\forall\, x \;\; \exists\, y \;\; x + y = 0$ is true.

Indeed, for $x = 2$, there exists y, namely $y = -2$, such that $\underbrace{x}_{2} + \underbrace{y}_{-2} = 0$.

All quantified sentences

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No matter which x is chosen, we may always find y, namely, $y = -x$, such that $x + \underbrace{y}_{-x} = 0$

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The statement $\forall\, x \;\; \exists\, y \;\; x + y = 0$ **is true.**

Indeed, for $x = 2$**, there exists** y**, namely** $y = -2$**, such that** $\underbrace{x}_{2} + \underbrace{y}_{-2} = 0$**.**

Why this reasoning is incorrect?

All quantified sentences

3) $\forall\, x \;\; \exists\, y \;\;\; x + y = 0$ T Why?

No matter which x is chosen, we may always find y, namely, $y = -x$,
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The statement $\forall\, x \;\; \exists\, y \;\;\; x + y = 0$ **is true.**

Indeed, for $x = 2$**, there exists** y**, namely** $y = -2$**, such that** $\underbrace{x}_{2} + \underbrace{y}_{-2} = 0$**.**

Why this reasoning is incorrect? Since x stands under universal quantifier,

All quantified sentences

3) $\forall\, x \;\; \exists\, y \;\; x + y = 0$ T Why?

No matter which x is chosen, we may always find y, namely, $y = -x$,
such that $x + \underbrace{y}_{-x} = 0$

☡ **Warning.** Here is a typical example of **incorrect** reasoning.

The statement $\forall\, x \;\; \exists\, y \;\; x + y = 0$ **is true.**

Indeed, for $x = 2$**, there exists** y**, namely** $y = -2$**, such that** $\underbrace{x}_{2} + \underbrace{y}_{-2} = 0$**.**

Why this reasoning is incorrect? Since x stands under universal quantifier,
one can not assign any value for it.

All quantified sentences

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An effective way to prove that a statement is false

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An effective way to prove that a statement is false is to construct its useful denial and show that the denial it true.

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**For any real number a, there exists a real number x
that satisfies the equation $ax^2 + x - a = 0$.**

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To solve the problem means to show that the statement above is true, that is,
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This is a quadratic equation (if $a \neq 0$) whose discriminant $1 + 4a^2$ is positive.

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So there is a solution (even two solutions).

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So there is a solution (even two solutions).

If $a = 0$, then the equation turns into $x = 0$.

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This is a quadratic equation (if $a \neq 0$) whose discriminant $1 + 4a^2$ is positive.
So there is a solution (even two solutions).

If $a = 0$, then the equation turns into $x = 0$. So there is a solution in this case also.

Hidden quantifiers

Consider the following problem:

Let a be a real number. Prove that the equation $ax^2 + x - a = 0$ has a real solution.

Although the problem contains no explicit quantifiers,
expressing the statement symbolically reveals the presence of two:

$$\forall a \in \mathbb{R} \ \exists x \in \mathbb{R} \quad ax^2 + x - a = 0 \,.$$

Here's a rewording that shows the quantifiers explicitly:

For any real number a, there exists a real number x that satisfies the equation $ax^2 + x - a = 0$.

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- construct useful denials of quantified sentences.

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The negation of the statement is therefore false.

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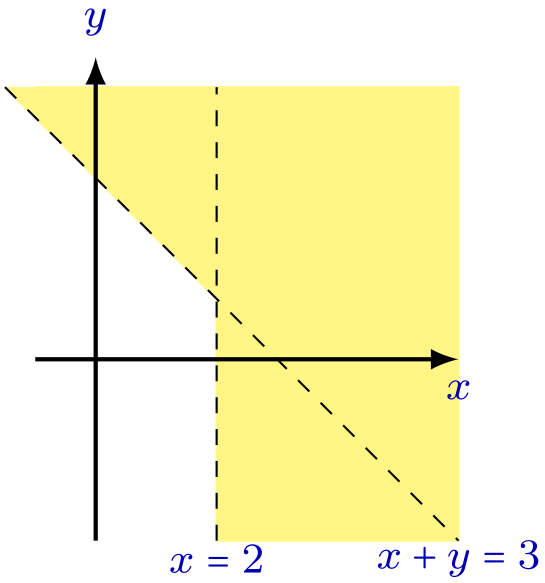

Geometric interpretation

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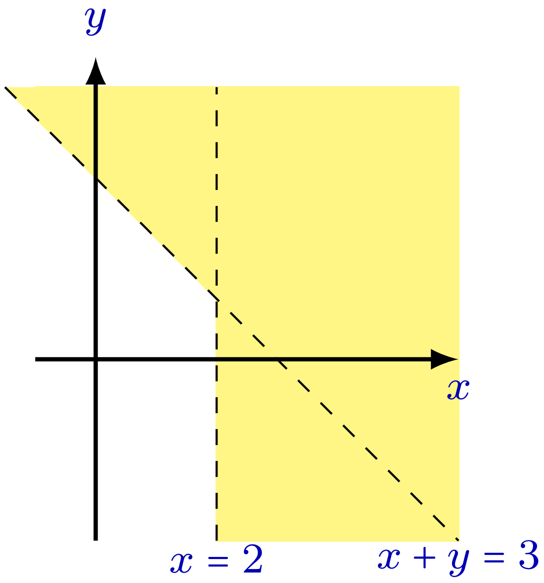

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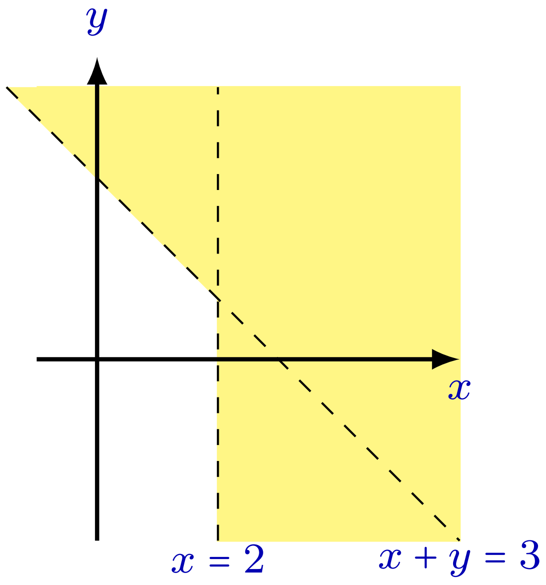

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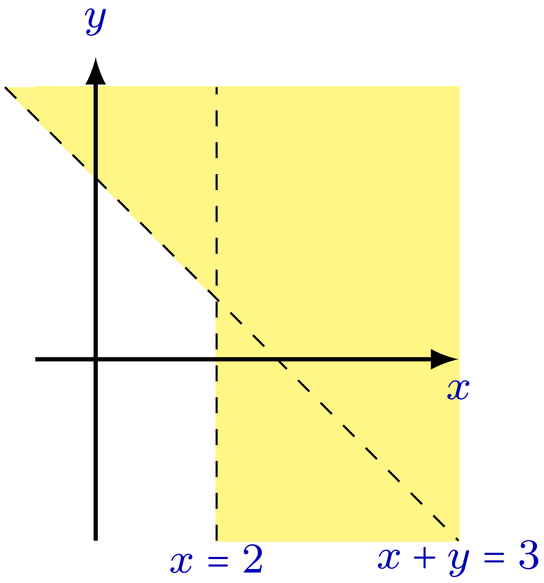

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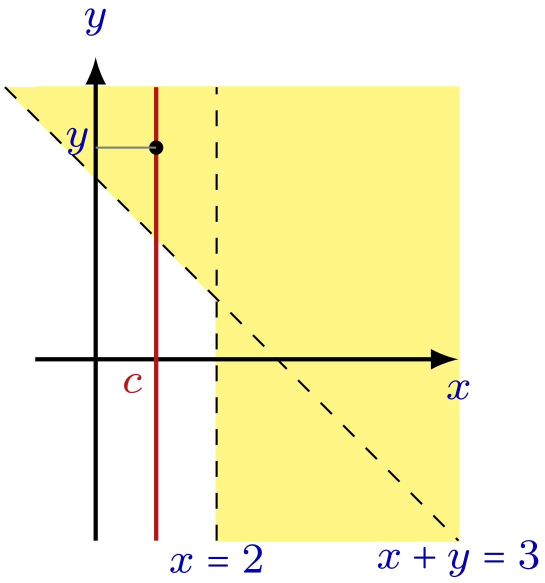

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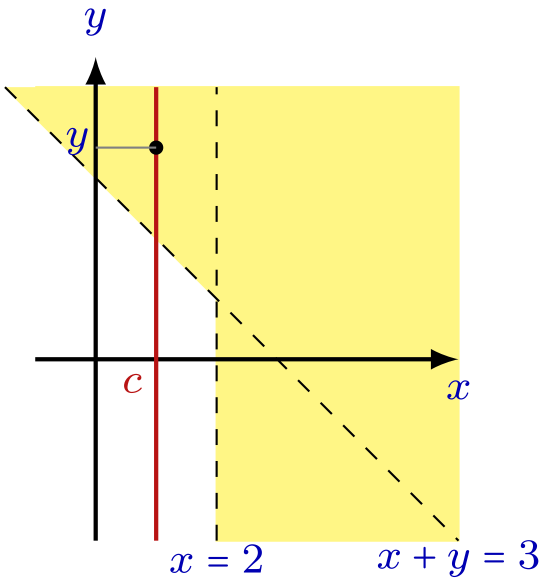

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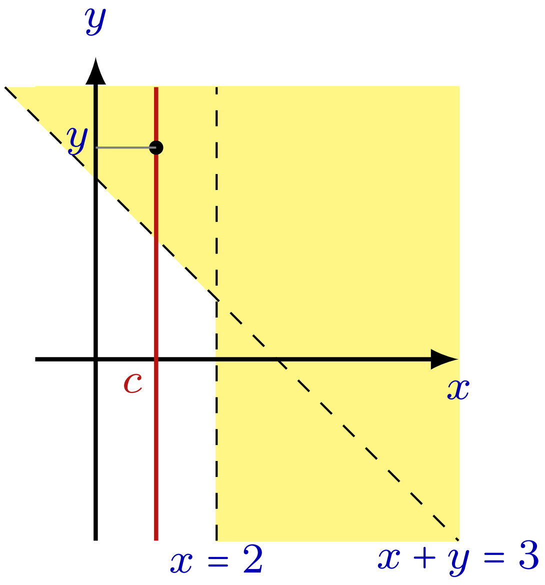

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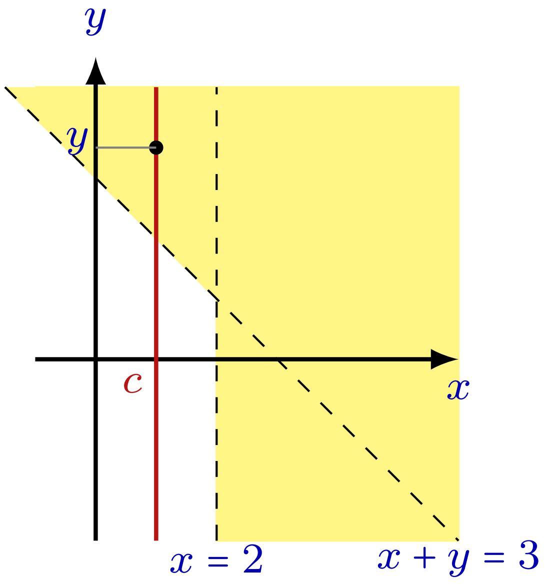

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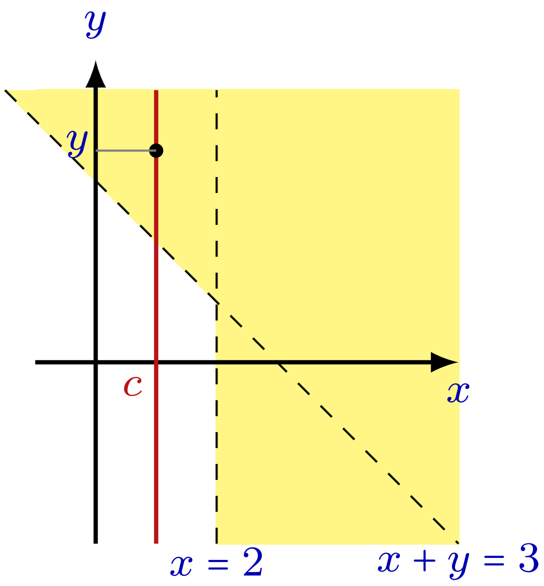

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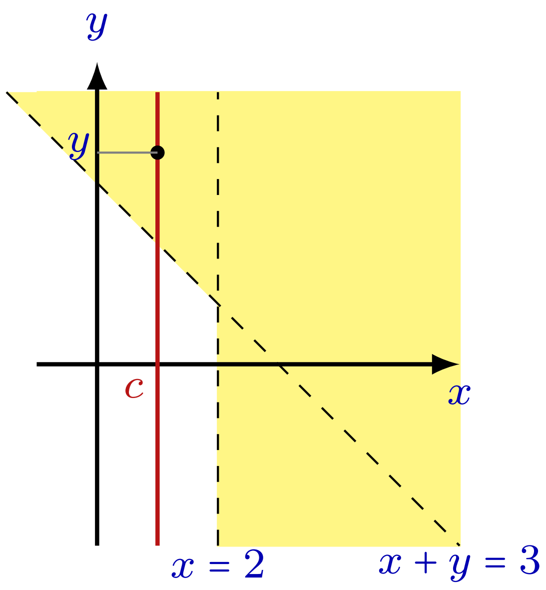

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A problem with parameter

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Problem. Let a and x be real numbers.

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So we have found all values of a such that $\exists\, x \;\; a^2x - x + 1 = 0$.

Answer: $a \neq \pm 1$.

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Can this happen for whatever a?

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The denial is true: no matter what a is, there is x, namely $x = 0$,

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Therefore, the denial is true for any a,

so the original statement is false for any a.

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$$a^2x - x + 1 = 0\,.$$

Can this happen for whatever a? No, since

$a^2x - x + 1 = 0 \iff (a^2 - 1)x + 1 = 0\,.$

To make our reasoning clear, let us consider a **denial** of the original statement.

$\neg(\forall\, x \;\; a^2x - x + 1 = 0) \iff \exists\, x \;\; a^2x - x + 1 \neq 0\,.$

The denial is true: no matter what a is, there is x, namely $x = 0$,

such that $a^2x - x + 1\Big|_{x=0} = 1 \neq 0\,.$

Therefore, the denial is true for any a,

so the original statement is false for any a.

Answer: There no such values of a.

A problem with parameter

In **(c)**, we need to find **all** values of a for which the sentence

$$\exists\, x \;\; a^2x - x + 1 > 0$$

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Answer: For any $a \in \mathbb{R}$, the statement hols true.