MAT131 Review for the final

1. Evaluate the integral

   a. $\int_1^8 (t^{2/3} - 2t^{4/3})\,dt$

   b. $\int_0^{\pi} (-\sin x + \cos x)\,dx$

   c. $\int_{-1}^{0} \frac{7}{\sqrt{1-x^2}}\,dx$

2. A rectangular playground is to be fenced off and divided into two by another fence parallel to one side of the playground. Six hundred feet of fencing is used. Find the dimension of the playground that will enclose the greatest total area.
3. For the function f(x)=x/(1+x)
   a. Find the vertical and horizontal asymptotes
   b. Find the intervals of increase or decrease.
   c. Find the local maximum and minimum values
   d. Find the intervals of concavity and the inflexion points
   e. Use the information from parts a. to d., to sketch the graph of f.
4. Find the linear approximation of the function $f(x)=(2+x)^{3/2}$ at x=2 and use it to approximate the numbers $(4.1)^{3/2}$, $(3.9)^{3/2}$ and f(2.2).
5. Let f be a function such that f(1)=-3 and $f'(x)=(2+x^3)^{-1}$. Use linear approximation to estimate the value of f(1.3).

6. Differentiate

$a.\ y = \dfrac{e^x \cos x}{3x \sin x}$

$b.\ y = \cos(\ln x)$

$c.\ y = \ln(\cos x)$

$d.\ y = 3^x$

$e.\ y = \tan^{-1}(2x - 4)$

$f.\ y = (\sin(x^2))^{3x}$

$g.\ y = \sqrt[3]{\dfrac{x^2 + 2x}{x^5 - 27}}$

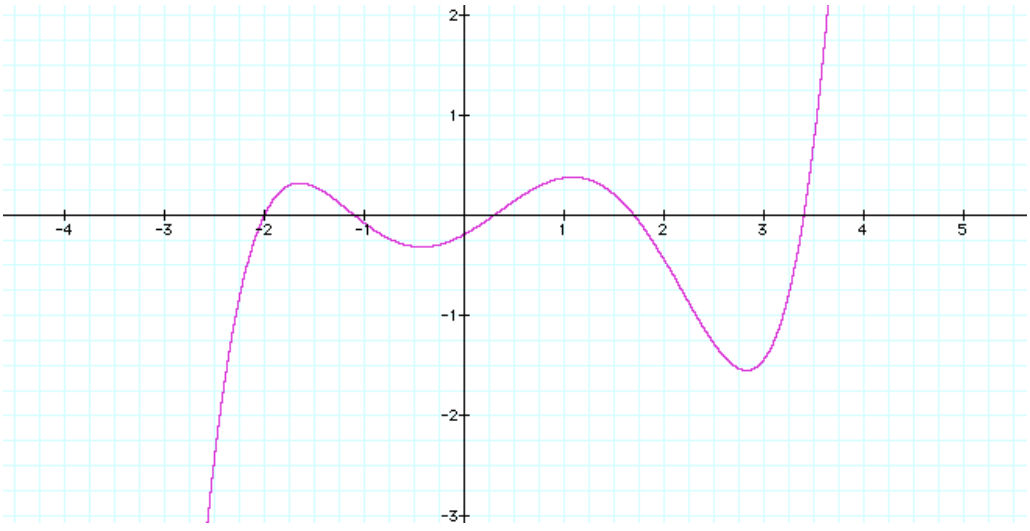
$h.\ y = |x|$

$i.\ y = \ln|x|$

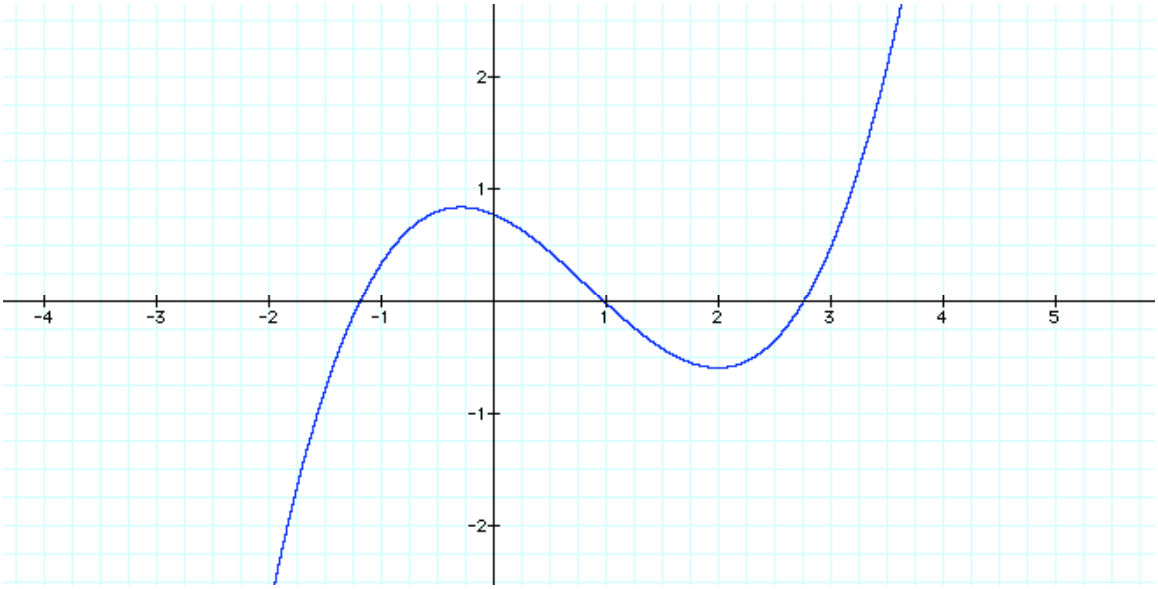
$j.\ y = \sin(\ln|x|)$

7. Consider the curve $\sqrt{x}+\sqrt{y}=3$
   a. Find y' and y'' by implicit differentiation.
   b. Find an equation of the tangent line at the point $(2,(3-\sqrt{2})^2)$
   c. Find all the points on the curve where the tangent has slope –1/2.

8. The graph of f is given. Sketch a graph of the derivative of f.



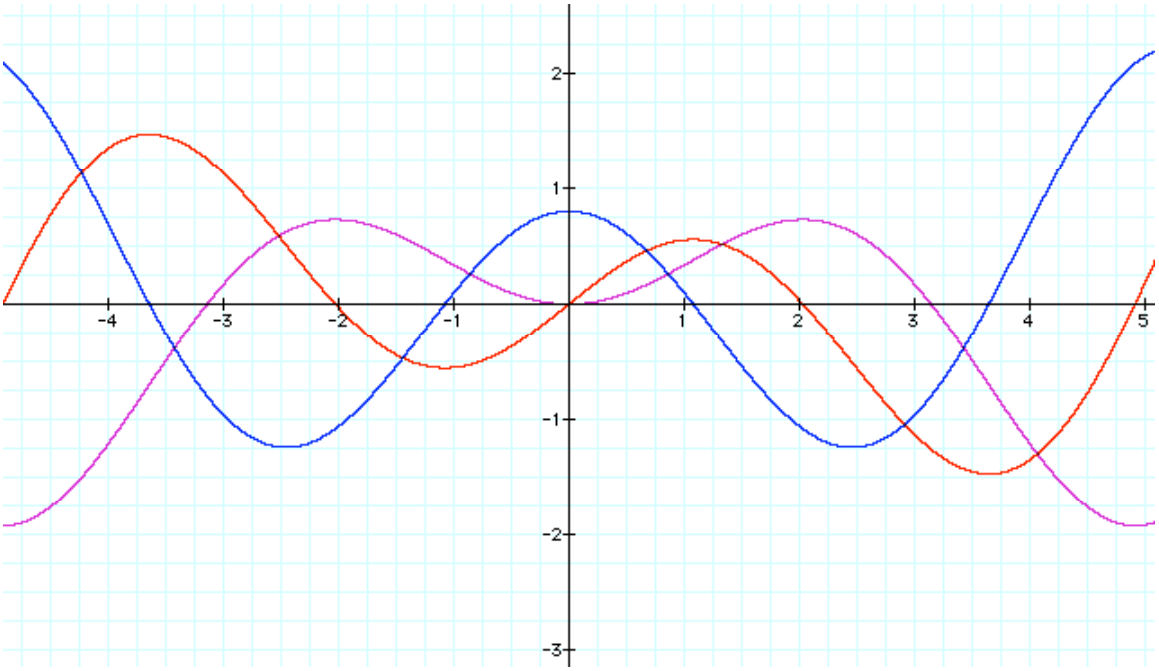
9. The graph of the derivative of f, is given. Sketch the graph of f if f(3)=0.5.



10. Sketch the graph of a function G with the following properties

   a. The domain of G is the interval [-4,5]
   b. G(0)=1 G'(0)=1
   c. G(1.2)=1.7, (1 point) G'(1.2)=0
   d. G'(x)>0 on (-2.5,1.2)
   e. G''(x)<0 on (-0.8,4]

11. Determine which is f, f'' and f''



12. Evaluate the limit, if it exists. If you use the fact that the function is continuous to evaluate the limit (that is, to "plug in" the value), state it. If the limit it does not exist, explain why.

a. $\lim_{x \to 3} e^{x^3 - 3x^2}$

b. $\lim_{x \to -4} \dfrac{x^2 - 16}{x^2 - 7x + 12}$

c. $\lim_{x \to 10} \dfrac{3 - \sqrt{x - 1}}{x - 10}$

d. $\lim_{x \to \infty} \dfrac{3x^2 - x + 2}{2x^2 + 5x^2 + 4}$

e $\lim_{x \to -\infty} \dfrac{x^5 + 4x}{-x^6 + 15x^2 + 41}$

f $\lim_{x \to -\infty} \dfrac{x^7 + 4x + 12}{-x^2 - 15x^3 + 31x}$

13. Consider the function $f(x) = \begin{cases} -x + 2 & \text{if } x < -3 \\ x^2 - c & \text{if } -3 \le x < 2 \\ x + 1 & \text{if } x \ge 2 \end{cases}$

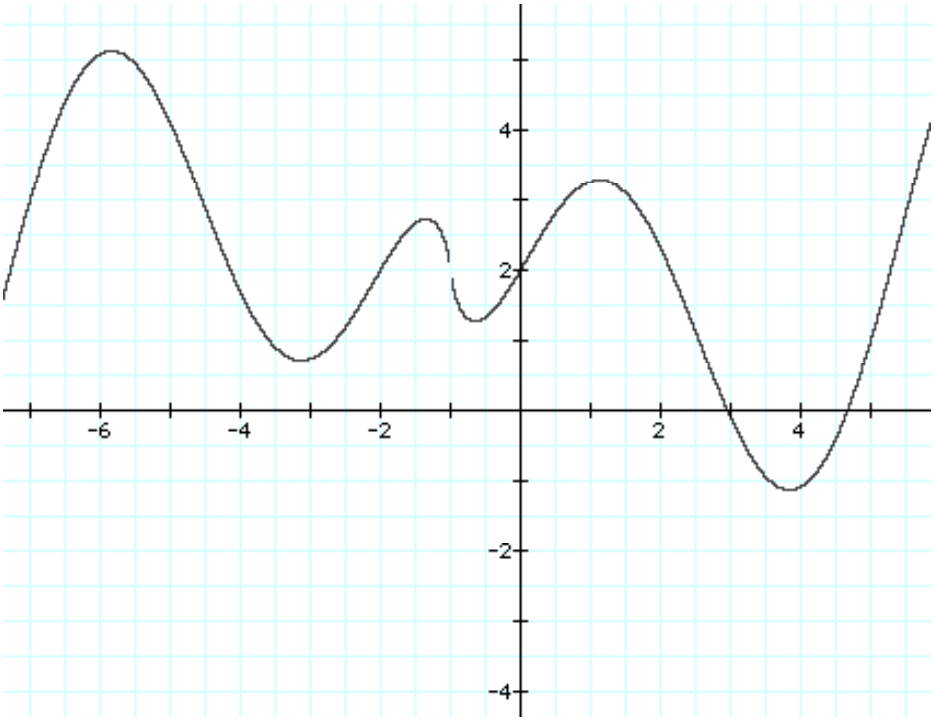
a. Sketch the graph of f for c=-2. Find the numbers at which f is discontinuous when c=-2.

b. Find the values of c for which f is continuous at all real numbers.

14. Use the intermediate value theorem to show that $f(x) = \frac{1}{2}x^3 - x^2 - x + 1$ the equation f(x)=0 has at least three solutions. For each of these solutions, find an interval of length at most 1 that contains it.

15. A particle moves with acceleration function $a(t) = 6 + 4t - 3t^2$. Its initial velocity is v(0)=2m/s ad its initial displacement is s(0)=5m. Find its position after 2 seconds.

16. The graph of f(x) is given.



a. Shade the area represented by $\int_{-4}^{2} f(x)dx$.

b. List the numbers $\int_{-4}^{2} f(x)dx$, $\int_{-6}^{4} f(x)dx$, *and* $\int_{4}^{5} f(x)dx$, in increasing order. Explain your reasoning.

c. Sketch a graph of a tangent line to the curve y=f(x) at x=4.

d. List the numbers f'(-6), f'(-4), and f'(-2) in increasing order. Explain your reasoning

It is strongly advisable that you solve more exercises than the ones on these pages before the final, with emphasis in the ones that you have more difficulty.

You can find more exercises in the book. Also, in the following webpages, you will find all kind of solved problems. http://www.math.ucdavis.edu/~kouba/ProblemsList.html

In particular, in http://www.math.ucdavis.edu/~kouba/CalcOneDIRECTORY/maxmindirectory/MaxMin.html you'll find solved problems about optimization. In

http://www.math.ucdavis.edu/~kouba/CalcOneDIRECTORY/graphingdirectory/Graphing.html you'll find solved problems about graphing functions using first and second derivatives.

Another link with exercises is: http://archives.math.utk.edu/visual.calculus/