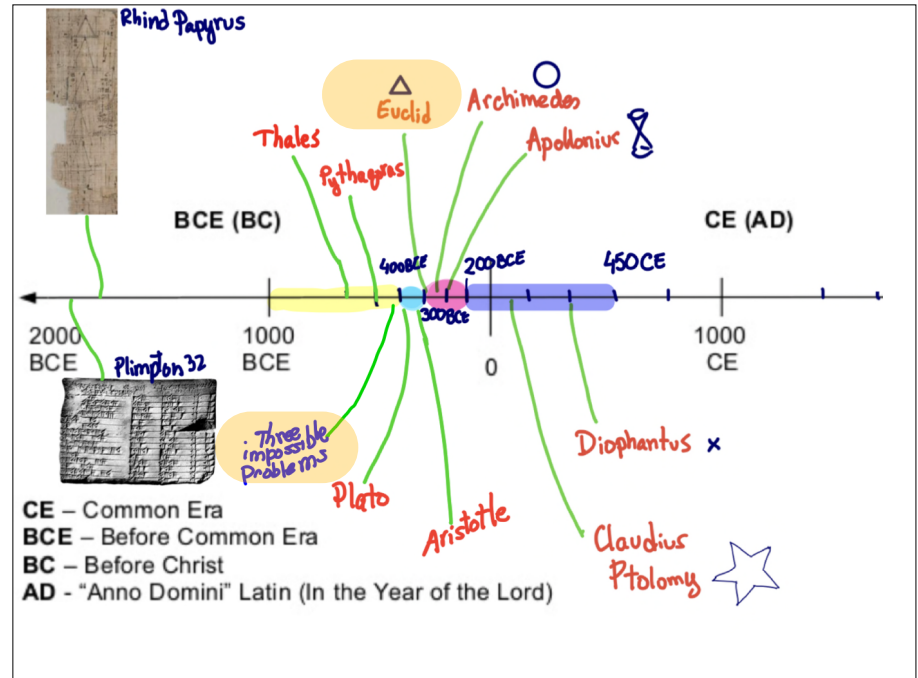


- Apollonius and conic sections
- Ptolemy and his table of chords.
- Archimedes
- Eratosthenes and the measurement of the Earth
- Diophantus and Algebra

MAT 336 Hellenic Mathematics After Euclid



Key words in Euclid's Elements

Teams

What do you remember about *Euclid's Elements*?
 What do you think are its most important ideas or methods?

Hints:

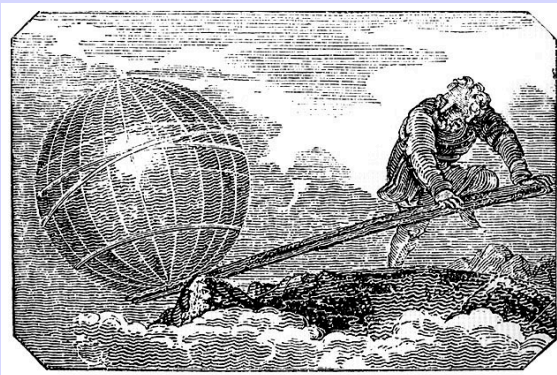
- Think of the *structure* — how knowledge is organized.
- Think of *proofs* — what kind of reasoning is used.
- Think of definitions, postulates, and common notions.
- How does Euclid start — from what kinds of statements?



Birds are fed by their parents in their infancy. When the time comes to feed themselves, there can be some confusion when the food does not go into their mouth by itself.
https://twitter.com/fasc1nate/status/1582504199015002112?s=48&t=6rkCZlkmqBgzNYl_WQLd4Q

Archimedes

Archimedes (~250 BCE)

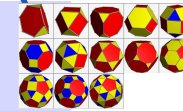


**GIVE ME A PLACE TO STAND
AND I WILL MOVE THE EARTH**

Archimedes (~250 BCE)

Derived and computed approximations to:

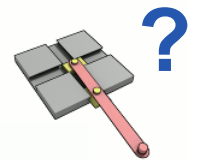
- Area of the circle
- Surface area and volume of the sphere
- Area of an ellipse.
- Area under a parabola



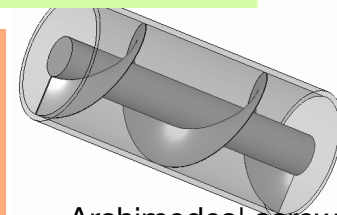
Ideas of calculus

- Infinitesimals
 - Method of exhaustion
- Dealt with infinity!!!

- Approximation of π
- Created a number system to deal with **arbitrary** large numbers

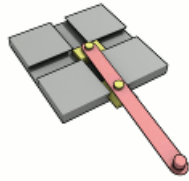
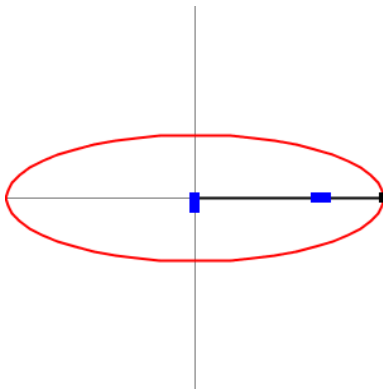


- Applied mathematics to physics
- Explanation of principle of the lever
- Innovative machines



Archimedes' screw ?

Trammel of Archimedes



Animation of the trammel of Archimedes by Alastair Rae for Wikimedia Commons

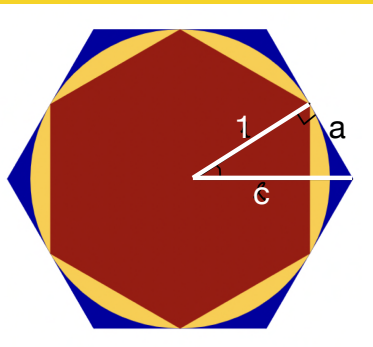
Archimedes and π

Archimedes' Method for Approximating π

A circle has **radius 1**.

Two regular hexagons are drawn:

- **Inscribed hexagon:** vertices on the circle
- **Circumscribed hexagon:** sides tangent to the circle



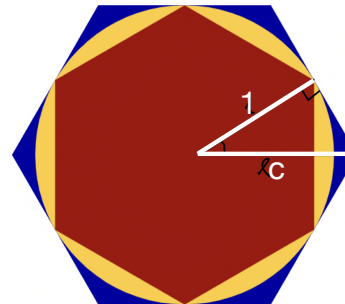
1. What is the perimeter of the inscribed hexagon?
2. What is the perimeter of the circumscribed hexagon? (Hint: What is the relation between a and c ?)

Archimedes' Method for Approximating π

A circle has **radius 1**.

Two regular hexagons are drawn:

- **Inscribed hexagon:** vertices on the circle
- **Circumscribed hexagon:** sides tangent to the circle



1. What is the perimeter of the inscribed hexagon?
2. What is the perimeter of the circumscribed hexagon? (Hint: study the angles in the three triangles in the figure below)

$$\left(\frac{c}{2}\right)^2 + 1^2 = c^2$$

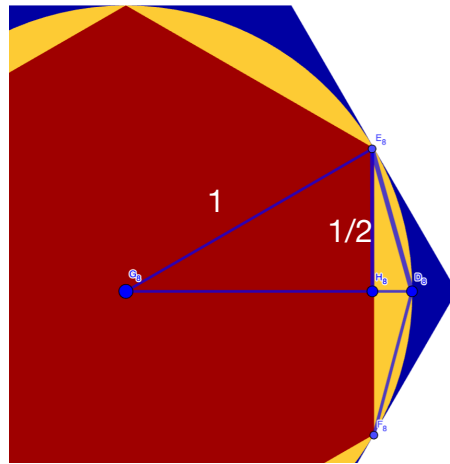
$$c^2 = \frac{3}{4}$$

Archimedes ~300BCE and the method of exhaustion

Method of exhaustion,
idea: Repeated subdivision
 can make a remainder
 smaller than any assigned
 magnitude



<https://www.geogebra.org/m/ajamkye2>



Archimedes' Bounds for π (radius = 1)

Idea: Computed the perimeter of inscribed and circumscribed regular n-gons

$$n = 6 \rightarrow 12 \rightarrow 24 \rightarrow 48 \rightarrow 96$$

Each doubling narrows the bounds for π

Used 96-sided polygons for his final bounds.

Archimedes Proposition: The ratio of the circumference of any circle to its diameter as less than $3 \frac{1}{7}$ but greater than $3 \frac{10}{71}$.

In symbols

$$3 \frac{10}{71} < \frac{\text{circumference}}{\text{diameter}} < 3 \frac{1}{7}$$

Educated guess:
Why did Archimedes
stop at 96 sides?
Why not 192 or 384?

Archimedes approximation of π

units	α	β	γ	δ	ϵ	ς	ζ	η	θ
	1	2	3	4	5	6	7	8	9
tens	ι	κ	λ	μ	ν	ξ	\omicron	π	υ
	10	20	30	40	50	60	70	80	90
hundreds	ρ	σ	τ	υ	ϕ	χ	ψ	ω	\aleph
	100	200	300	400	500	600	700	800	900
thousands	α	β	γ	δ	ϵ	ς	ζ	η	θ
	1000	2000	3000	4000	5000	6000	7000	8000	9000

Source: Wikimedia

Archimedes' Bounds for π (radius = 1)

n	Inscribed	Circumscribed
6	3	$2 \cdot 3^{1/2}$
12	$(2 - 3^{1/2})^{1/2} \times 12$	$2 \cdot 3^{1/2} / (1 + 3^{1/2}) \times 12$
24	$(2 - (2 + 3^{1/2})^{1/2})^{1/2} \times 24$	$2 \cdot (2 - 3^{1/2})^{1/2} \cdot (2 \cdot 3^{1/2} / (1 + 3^{1/2})) / ((2 - 3^{1/2})^{1/2} + 2 \cdot 3^{1/2} / (1 + 3^{1/2})) \times 24$
48	$(2 - (2 + (2 + 3^{1/2})^{1/2})^{1/2})^{1/2} \times 48$	$2 \cdot a_{24} \cdot b_{24} / (a_{24} + b_{24}) \times 48$
96	$3 \frac{10}{71}$	$3 \frac{1}{7}$

$$a_{2n} = (2 - (4 - a_n^2)^{1/2})^{1/2}$$

$$b_{2n} = 2 \cdot a_n \cdot b_n / (a_n + b_n)$$

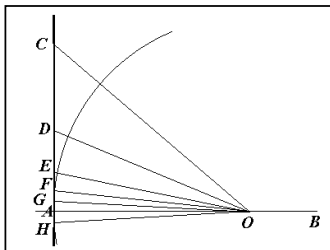
Archimedes' Bounds for π (radius = 1)

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96	$3 \frac{10}{71}$	$3 \frac{1}{7}$

Archimedes' Proposition: The ratio of the circumference of any circle to its diameter is less than $3 \frac{1}{7}$ but greater than

$$3 \frac{10}{71} < \frac{\text{circumference}}{\text{diameter}} < 3 \frac{1}{7}$$

Archimedes approximation of π



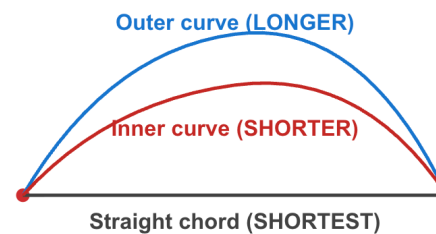
- O – center of the circle
- A, B – ends of the diameter (AB passes through O)
- AC – tangent at A
- C – point on the tangent where $\angle AOC = 30^\circ$ (circumscribed hexagon)
- D, E, F, G, H – successive intersections of the tangent AC with lines from O that bisect the previous central angles (12-, 24-, 48-, 96-gons), each nearer A.

What did Archimedes mean by the *length of a curve*?

Archimedes Ordering axioms (modern version).

Axiom 1: Of all lines which have the same extremities, the straight line is the least.

Axiom 2: Of other lines in a plane with the same endpoints and concave in the same direction, that which is **closer to the straight line** (or lies entirely between **the other line** and the **straight line**) is the shorter.

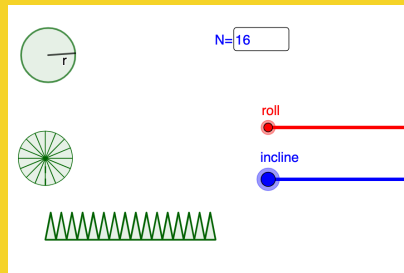


Archimedes treated the **length of a curve** as a **magnitude**—something you can compare (greater/equal/less) but not compute.

So how did Archimedes compute π ?

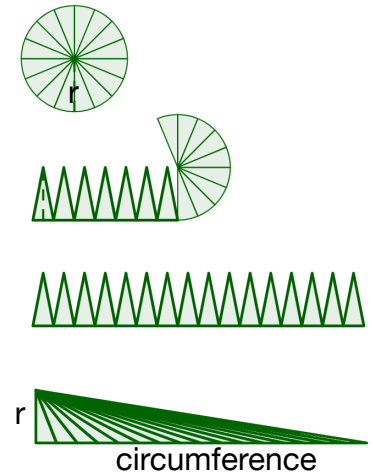
<https://www.geogebra.org/m/fg9d4ueb>

1. As the number of sectors grows, what are the lengths of the legs of the limiting right triangle?
2. The area of a circle with radius R equals the area of a right triangle with legs A and B . What are A and B ?



Archimedes (~250BCE) on the Measurement of a Circle

For I realized that just as every circle equals a triangle having as its base the circumference of the circle and altitude equal to the [distance] from the center to the circle [that is, the **radius**]...



This shows that the constants for computing circumference and area of the circle are the same, π .

Archimedes and large numbers

Archimedes The sand reckoner (~250BCE)

Created a number system to deal with **arbitrary** large numbers

Archimedes computed the **number of grains of sand that would fill a sphere whose diameter was equal to the distance from earth to the fixes stars.**

To work with very large numbers, he imagined a “doubled class” of numbers of eight numerals (instead of the four the the Greek ciphered system)

1. 1 to 99,999,999
2. 100,000,000 to $10^{16}-1$
3. etc

units	α	β	γ	δ	ϵ	ς	ζ	η	θ
	1	2	3	4	5	6	7	8	9
tens	ι	κ	λ	μ	ν	ξ	\omicron	π	ρ
	10	20	30	40	50	60	70	80	90
hundreds	σ	τ	υ	ϕ	χ	ψ	ω	ϑ	
	100	200	300	400	500	600	700	800	900
thousands	α	β	γ	δ	ϵ	ς	ζ	η	θ
	1000	2000	3000	4000	5000	6000	7000	8000	9000

Wikimedia

Some of the numbers to which I have given a name [...] surpass not only the number of grains of sand that could fill the Earth [...] but even the number of grains of sand that could fill the universe itself.

Archimedes, Sand-Reckoner

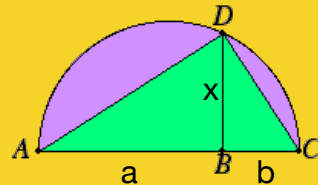
Mean Proportional

Proposition VI.16 of Euclid's elements

To find a mean proportional to two given segments.

Prove that BD is the mean proportional between AB and BC. In other words, show that $AB : BD = BD : BC$.

A positive number x is the **mean proportional** of two (positive) numbers a and b if $a/x=x/b$.



Join the lesson at www.geogebra.org/classroom with the code:

JEUTQGMJ

Or you can also share the following link with your students:

www.geogebra.org/classroom/jeutqgmj

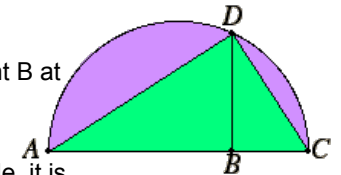


<https://www.geogebra.org/classroom/jeutqgmj>

Proposition VI.16 of Euclid's elements

To find a mean proportional to two given straight lines.

- Let AB and BC be the two given straight lines.
- It is required to find a mean proportional to AB and BC.
- Place them in a straight line, and describe the semicircle ADC on AC. Draw BD from the point B at right angles to the straight line AC, and join AD and DC. (I.11)
- Since the angle ADC is an angle in a semicircle, it is right. (III.31)
- Since, in the right-angled triangle ADC, BD has been drawn from the right angle perpendicular to the base, therefore BD is a mean proportional between the segments of the base, AB and BC. (VI.8, Corollary)
- **Therefore a mean proportional BD has been found to the two given straight lines AB and BC**



A positive number x is the **mean proportional** of two (positive) numbers a and b if $a/x=x/b$.

From mean proportional to conic sections

- **Hippocrates of Chios (~400 BCE)** reduced the problem of **doubling the cube** to finding **two mean proportionals** between two given lengths a and b .

In modern terms, the unknowns x and y satisfy: $a : x = x : y = y : b$, or equivalently:

1. $x^2 = ay$
2. $y^2 = bx$
3. $xy = ab$.

- **Menaechmus (~350 BCE)** discovered **conic sections** while seeking a geometric **construction** of these two mean proportionals.

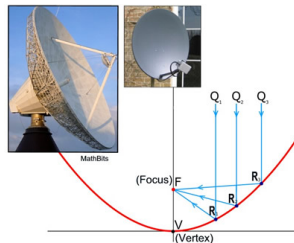
Doubling the cube → Two mean proportionals → Conic sections

Apollonius of Perga (~200 BCE) Studied parabolas.

Current applications of properties parabolas

A parabolic dish is a surface with a cross-sectional shape of a parabola -paraboloid- used to direct light or sound waves. This concept is used by

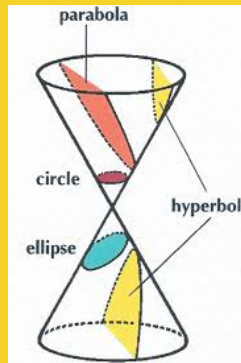
- Radio telescope antennas
- satellite dishes
- parabolic reflector,
- car's headlights and in spotlights.



<https://mathbitsnotebook.com/Algebra2/Quadratics/QDPParabolaApplied.htm>

Conic sections

<https://www.geogebra.org/m/pgu4vvtj>



<http://jwilson.coe.edu/em1725/Conics%20Images/Conics.html>

1. When $\alpha = 40^\circ$ and $\beta = 35^\circ$, the intersection is a hyperbola.

Find all pairs (α, β) that produce a hyperbola.

2. For which values of (α, β) is the intersection an ellipse?

3. For which values of (α, β) is the intersection a circle?

4. For which values of (α, β) is the intersection a parabola?

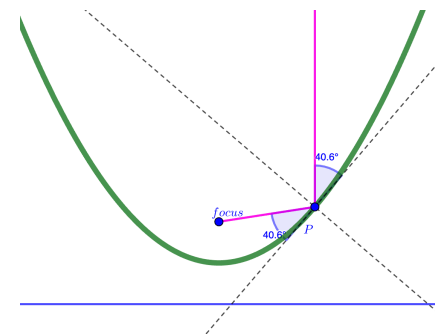
5. Can you find any other type of curve formed by this intersection?

Compare the angle of the plane with the cone's axis (α) to the angle of the cone's side with the axis (β)

Find all the values you can — the GeoGebra app will help you visualize and test your answers.

How a Parabolic Mirror Focuses Light

<https://www.geogebra.org/m/fkqe4v5m>



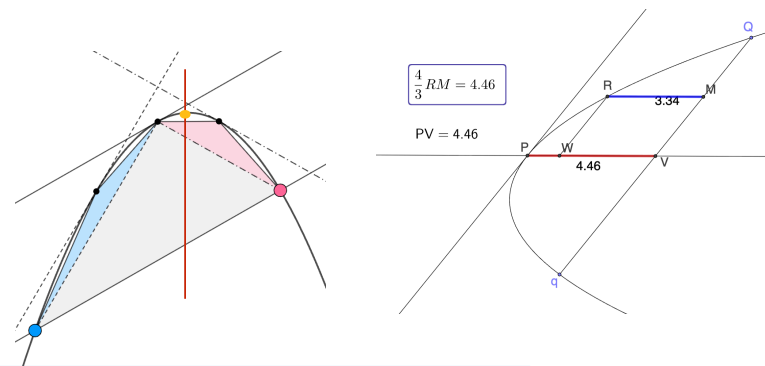
- <https://www.geogebra.org/m/jeqqdnkh> (spherical mirror for comparison)

Archimedes and the area under the parabola

Archimedes Quadrature of the Parabola

<https://www.geogebra.org/m/rmfkggwg>

<https://www.geogebra.org/m/re5mq6jt>



The area of the parabolic segment equals $\frac{4}{3} \times$ (the area of the triangle with the same base and height) —the first known exact “quadrature” of a curved figure.

He proved (in modern notation)
Vertex (0,0)
y is proportional to x^2

Stories about Archimedes

Eureka!





Death of Archimedes (Mosaic, Municipal Gallery Frankfurt)

Archimedes last words



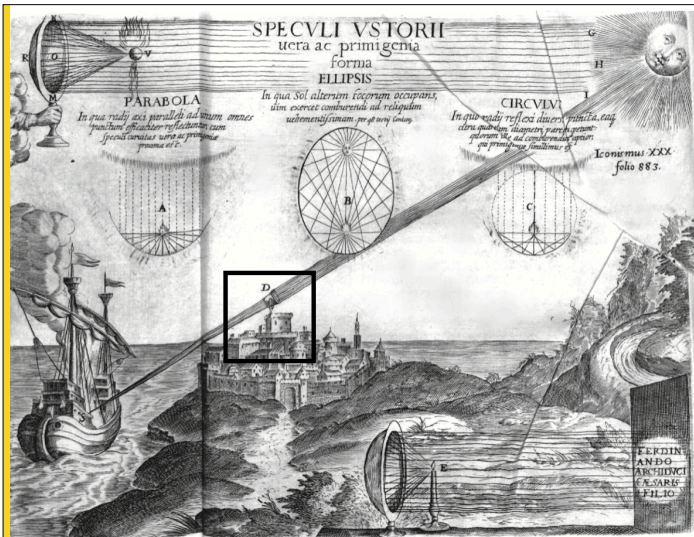
Cassus: "The head and not the lines"

Georgios Pachymeris: "heat me (my head) do not destroy the lines
"(drawing)

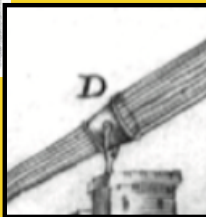
Tzetzes: "Stay away man from my diagram"

Unknown: "Don't disturb my circles"

A story about Archimedes' and mirrors

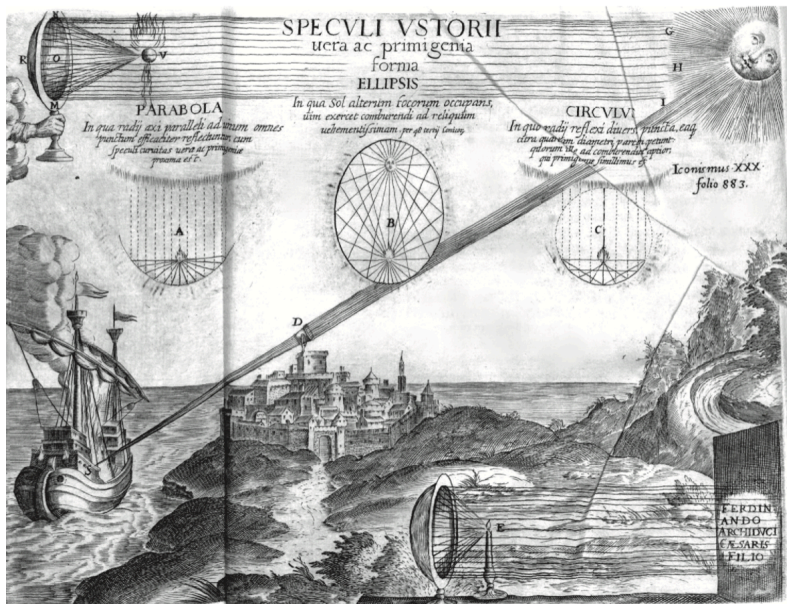


What's happening at point D? Why might this be important?



Archimedes' mirror





Setting fire to a ship using the sun and a mirror. Engraving from a book by our buddy Athanasius Kircher, 1671. (Wellcome Collection)

One of the many stories about Archimedes tells that he managed to destroy the Roman fleet at the siege of Syracuse in 213 BC by the application of directed solar radiant heat.

Many studies debunked this...

The Archimedes' Palimpsest

The Archimedes' Palimpsest

- **10th century:** Scribe in Constantinople copied Archimedes texts
- **13th century:** Christian monks needed vellum for a prayer book — washed off Archimedes, wrote prayers over it
- Named **Palimpsest** (manuscript with text written over earlier, erased text)
- **Discovered 1906** in a monastery in Constantinople. **reappeared in the 1970's**, (in the hands of a French family that had bought it in Istanbul in the 1920s) **resurfaced** at Christie's auction **1998** (badly damaged)
- Anonymous collector bought for **\$2m** → deposited at **Baltimore's Walters Art Museum**
- **Importance:** Revealed *The Method* — Archimedes' techniques for discovering geometric results using physical reasoning before formal proof.

Vellum = writing material made from treated calfskin; expensive and reusable

The Stomachion: Archimedes' Forgotten Work

A puzzle well known in Antiquity: 14 irregular pieces
→ how many ways to form a square?

Archimedes presents systematic ideas of counting all these ways

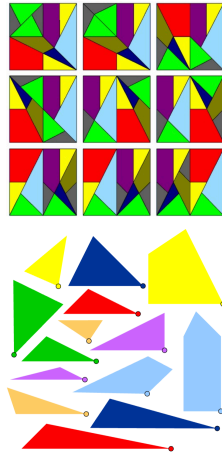
2006 answer to Archimedes' question (interpreted): 17,152 configurations (required systematic counting of *all* possibilities)

Why ignored? Only references until the discovery of the Palimpsest.

Why important? A treatise on **combinatorics** — a field that didn't develop until the rise of **computer science**

Far ahead of its time

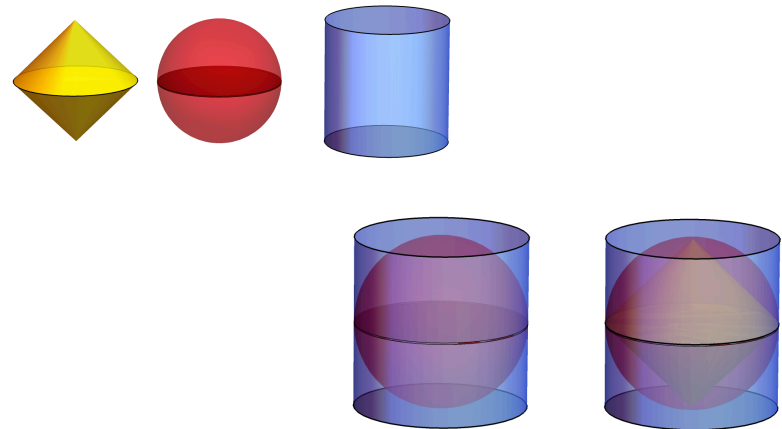
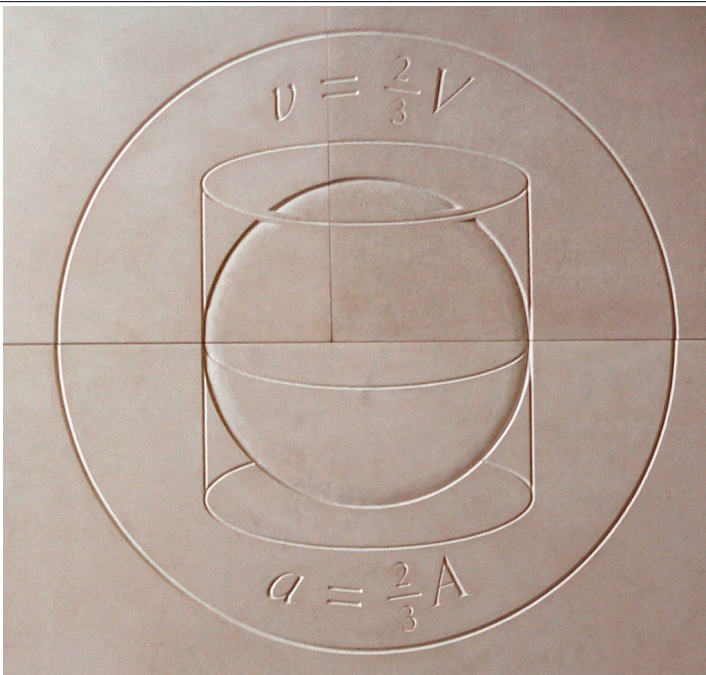
Challenge: How many ways can *you* arrange these pieces into a square?



Archimedes' favorite result

<https://www.geogebra.org/m/d9thyaxv>

<https://pi.math.cornell.edu/~mec/GeometricDissections/1.2%20Archimedes%20Stomachion.html>



1. The horizontal *green* plane cuts the **sphere** of radius r at height h , forming a **circle**. Write the **area of this circle** in terms of r and h .

2. The same plane cuts the cylinder with a cone removed, forming an annulus (ring). Write the area of this annulus in terms of r and h .

3. Compare your two expressions.

What relationship do you find between the **areas** of the sphere's cross-section and the ring's cross-section?

What does this imply about the **volumes** of the sphere and the cylinder-cone solid?

C = volume of cylinder (height r , base r)

S = volume of sphere (radius r)

O = volume of cone (height r , base r) = $C/3$

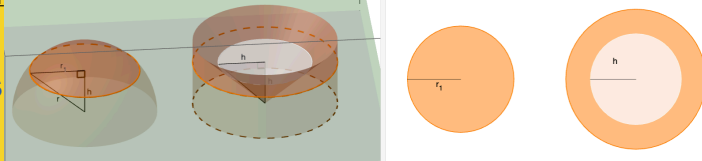


[https://](https://www.geogebra.org/m/dnv3fph3)

[www.geogebra.org](https://www.geogebra.org/m/dnv3fph3)

[g/m/dnv3fph3](https://www.geogebra.org/m/dnv3fph3)

Archimedes (~250BCE)



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S = volume of sphere (radius r)

O = volume of cone (height r , base r) = $C/3$

$S = C - O = \frac{2}{3}C$.

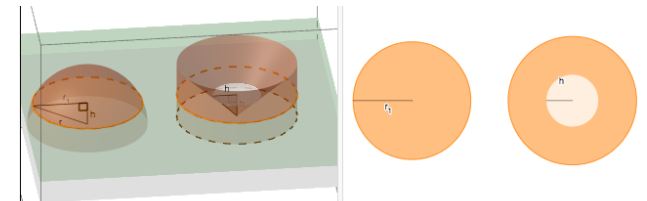


[https://](https://www.geogebra.org/m/dnv3fph3)

[www.geogebra.org](https://www.geogebra.org/m/dnv3fph3)

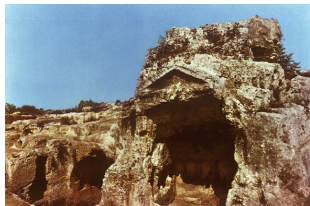
[g/m/dnv3fph3](https://www.geogebra.org/m/dnv3fph3)

Archimedes (~250BCE)



And although he made many excellent discoveries, he is said to have asked his kinsmen and friends to place over the grave where he should be buried a cylinder enclosing a sphere, with an inscription giving the proportion by which the containing solid exceeds the contained.

Plutarch (AD 45-120), Parallel Lives: Marcellus



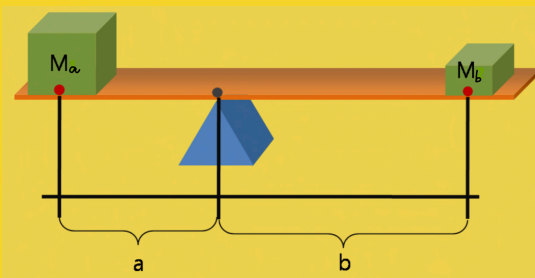
A tomb in Syracuse in the Necropolis of Grotticelli referred to affectionately (or deceptively) as "[Archimedes' Tomb](https://www.math.nyu.edu/~corres/Archimedes/Tomb/TombIllus.html)", but known to be of Roman origin dating at least two centuries after the death of Archimedes.

<https://www.math.nyu.edu/~corres/Archimedes/Tomb/TombIllus.html>

Archimedes and the Law of the Lever

Archimedes Law of the Lever

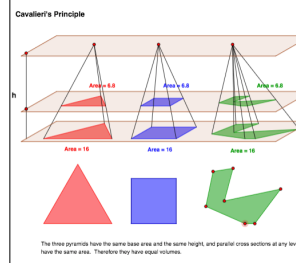
Magnitudes are in equilibrium at distances reciprocally proportional to their weights.



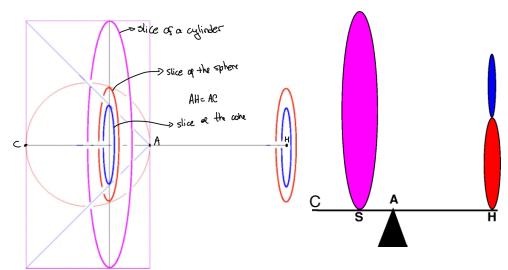
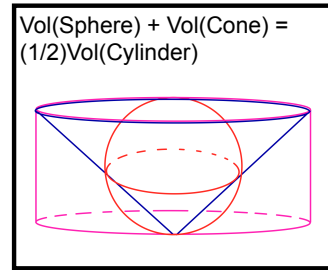
Write down a formula for Archimedes Law of the Lever in terms of the weights M_a and M_b and the distances a and b

Source: Wikimedia. https://commons.wikimedia.org/wiki/File:Lever_Principle_3D.png

Cavalieri's principle (although was known by Chinese mathematicians before Cavalieri stated it)



Two solids such that the section of one by each horizontal plane bears a fixed ratio to the section of the other by the same plane have volumes in that same ratio.



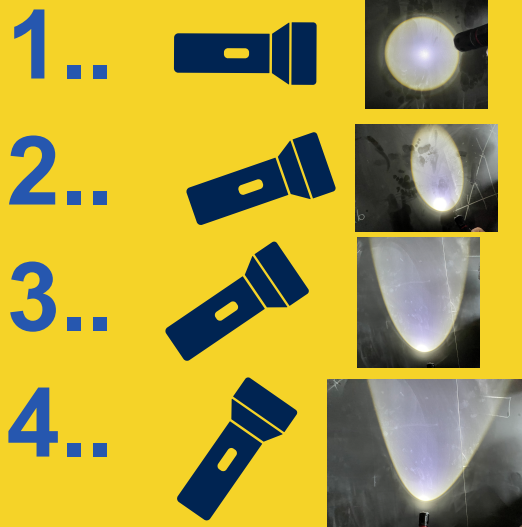
- Applied Cavalieri's principle, imagining the sections of a region balanced about a fulcrum
- If each pair of corresponding sections balance at distances a and b , then the bodies themselves will balance at these distances
- The balancing force of the cylinder is the same if all of its mass is concentrated at its center of mass, which is the midpoint of AC .

Example of Archimedes' Method

Archimedes Law of the Lever: Magnitudes are in equilibrium at distances reciprocally proportional to their weights.

Archimedes' Spiral

What curves do you see in the demonstration?
 Explain why we see those curves. (Hint: there should be a cone in the explanation)

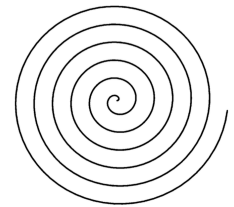


<https://www.geogebra.org/m/rVx87NXP>

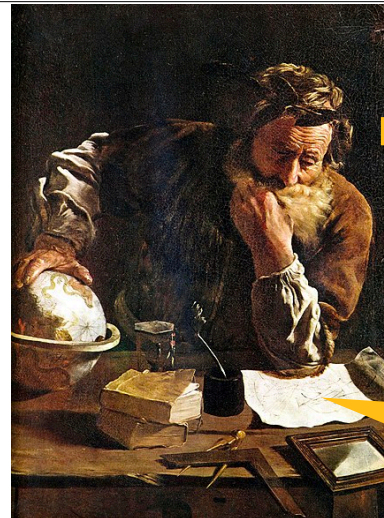
Archimedean spiral

If a straight line drawn in a plane revolve at a uniform rate about one extremity which remains fixed and return to the position from which it started, and if, at the same time as the line revolves, a point move at a uniform rate along the straight line beginning from the extremity which remains fixed, the point will describe a spiral (ελίξ) in the plane.

(ελίξ) Helix



Eratosthenes, the
 librarian of Alexandria
 (~200 BCE)



Archimedes Thoughtful (also known as Portrait of a Scholar) by Domenico Fetti, 1620



By Bernardo Strozzi - Musée des beaux-arts de Montréal, Public Domain, <https://commons.wikimedia.org/w/index.php?curid=42582581>

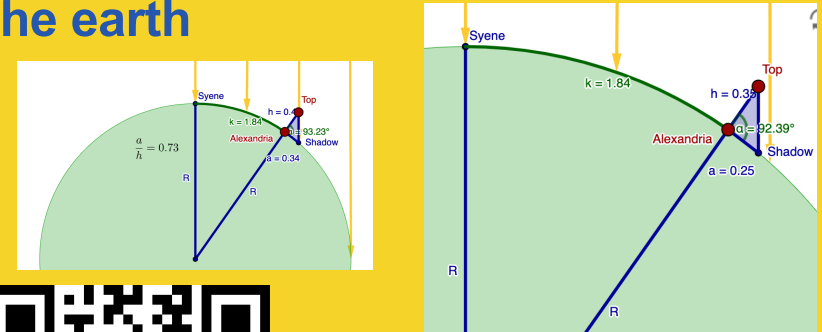
“The proofs then of these theorems I have written in this book and now send to you (that is, Eratosthenes)”

A well at Kom Ombo, 50 km north of Syene



Image credit: <https://medium.com/roaming-physicist/looking-down-a-well-in-egypt-to-measure-the-size-of-the-earth-5e12f2df0b2>

Eratosthenes on the circumference of the earth

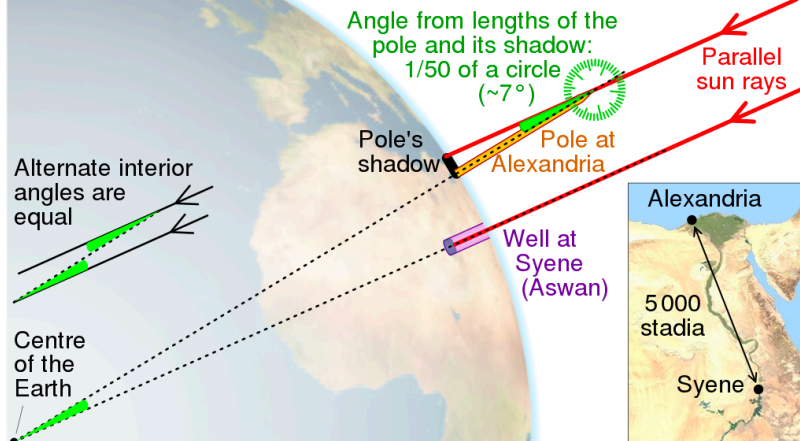


Discussion in groups.
Questions in Geogebra.
Answer individually in Wooclap.

<https://www.geogebra.org/m/ty5ych75>

Illustration of the method Eratosthenes used to calculate the circumference of the Earth

1/50 of a circle \leftrightarrow 5000 stadia (~800 km)
 \therefore 1 circle \leftrightarrow 50 \times 5000 stadia
 = 250 000 stadia (~40 000 km)

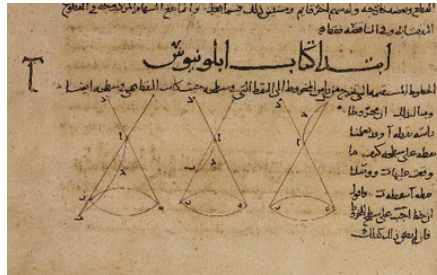


cmglee, David Monniaux, jimht at shaw dot ca, CC BY-SA 4.0 <<https://creativecommons.org/licenses/by-sa/4.0/>>, via Wikimedia Commons

Apollonius (~200 BCE)

Apollonius of Perga (~200BCE)

- Alexandria
- worked out more fully and generally than in the writings of others
- studied and later taught under the followers of Euclid



Conics - 8 books

"the most and prettiest of these theorems are new, and it was their discovery which made me aware that Euclid did not work out the syntheses of the locus with respect to three and four lines, but only a chance portion of it, and that not successfully; for it was not possible for the said synthesis to be completed without the aid of the additional theorems discovered by me."

Hypatia (~400 CE) was a philosopher, astronomer, and mathematician who lived in Alexandria.

In the clip below from movie Agora (a fictionalized version of her life), she starts by discussing the assumption that the orbits of the earth around the sun area circles.



Hypatia arrives to the conclusion that the orbits follow another curve and not a circle. What is that curve? What is the property of that curve that she describes?

<https://youtu.be/bUUB1u8xJA4>

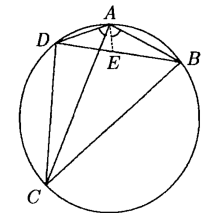
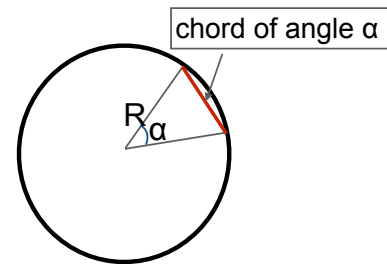
What can I do to make the lectures more interesting? Is there any special topic you would like to be covered? Maybe something closer to the present? Any other suggestion?

Claudius Ptolemy (100 – 170 CE)

Claudius Ptolemy ~100AD

Mathematical Collection (Almagest) - Mathematical Astronomy

- Astronomer and physicist (hence, mathematician of his era)
- Trigonometry - Table of chords
 - Solved plane triangles
 - Solved spherical triangles



Claudius Ptolemy ~100AD

I know that I am mortal and the creature of a day; but when I search out the massed wheeling circles of the stars, my feet no longer touch the Earth, but, side by side with Zeus himself, I take my fill of ambrosia, the food of the gods.

(Ptolemy's Almagest)

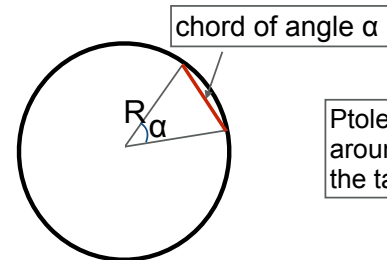


Claudius Ptolemy Table of Chords

The idea of **angles** seems to have appeared in Greece between 6th and 5th century BCE.

Trigonometry is the branch of mathematics that studies relationships between side lengths and angles of triangles.

Ptolemy made a table of correspondence between angles and their chords. (This is the earliest form of trigonometry we know of)



Ptolemy's treatise on astronomy, written around 150 CE, explains how to construct the table.

Claudius Ptolemy ~100AD

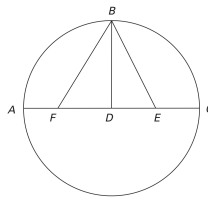
- Worked with a circle of radius 60 (because he did calculations in base 60) **Question: Why base 60?**

- Computed chord(36°). How?

Main idea: computed chord tables of certain angles using

- propositions from Euclid's elements
- square roots (or approximations)

In the Almagest, Ptolemy announces that he is going to employ in general the base 60 system, in order to avoid "the embarrassment of the fractions"



Example: Suppose that E bisects DC
 Find F so EF=EB
 Then (prove it!) DF is the side of a decagon inscribed on a circle with radius DC.
 Ptolemy computed DF=chord(36°):

$$DF^2 = (EF - ED)^2 = (EB - ED)^2 = ((BD^2 + ED^2)^{1/2} - ED)^2 = ((60^2 + 30^2)^{1/2} - 30)^2 = (37;4;55)_{60}$$

Claudius Ptolemy ~100AD

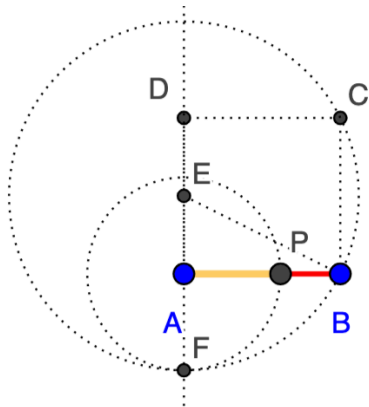
- regular dodecagon -> chord(30°)
- regular decagon -> chord(36°)
- chord(36°) and chord(30°) -> chord(6°)
- Bisecting chord(3°) -> chord(1° 30')
- Upper and lower values of the chord(1°)
- Finally, a table of 360 entries, with chords of at 30f' increments up to 180°.

περιμε- τρειών	εὐθειῶν			ἑξηκοσίων			
Λ'	ο	λα	κα	ο	α	β	ν
α	α	β	γ	ο	α	β	ν
αΛ'	α	λδ	εε	ο	α	β	ν
β	β	ε	μ	ο	α	β	ν
βΛ'	β	λς	δ	ο	α	β	μνη
γ	γ	η	καη	ο	α	β	μνη
γΛ'	γ	λθ	νβ	ο	α	β	μνη
δ	δ	ια	ις	ο	α	β	μς
δΛ'	δ	μβ	μ	ο	α	β	μς
ε	ε	ιδ	δ	ο	α	β	μς
εΛ'	ε	με	κς	ο	α	β	μς
ς	ς	ις	μθ	ο	α	β	μδ
ςΛ'	ς	μη	ια	ο	α	β	μγ
ζ	ζ	ιθ	λγ	ο	α	β	μβ
ζΛ'	ζ	ν	νθ	ο	α	β	μκ
η	η	κβ	ιε	ο	α	β	μκ
ηΛ'	η	νγ	λε	ο	α	β	λθ
θ	θ	κδ	νμ	ο	α	β	λη
θΛ'	θ	νς	ιγ	ο	α	β	λς
ι	ι	κς	λβ	ο	α	β	λε
ιΛ'	ι	νη	μθ	ο	α	β	λγ
ιαΛ'	ια	λ	ε	ο	α	β	λβ
ιβ	ιβ	α	κα	ο	α	β	λ
ιβΛ'	ιβ	λβ	λς	ο	α	β	καη

Beginning of Ptolemy table of chords

<https://www.wilbourhall.org/pdfs/HeibergAlmagestComplete.pdf>

Euclidean Construction of the Golden Ratio



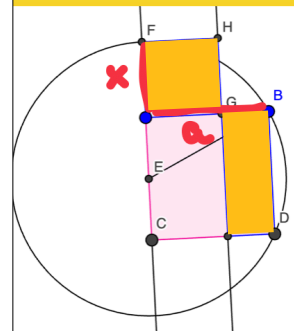
Two positive numbers a and b are in the **golden ratio** if $(a+b)/a = a/b$ and $a > b$
 or $(a+b)/b = b/a$ and $b > a$

<https://www.geogebra.org/m/fyassfgv>

[proof http://www.u.arizona.edu/~sreyes/archived/GoldenSection.htm](http://www.u.arizona.edu/~sreyes/archived/GoldenSection.htm)

A straight line (segment AB) is given.
 Set $a = AB$ and $AH = x$. Write down an equation relating a and x (Hint: x is the point such that the two yellow areas are equal.)

Proposition II.11: To cut a given straight line so that the rectangle contained by the whole and one of the segments equals [the area of] the square on the remaining segment.



Extract from Euclid's "Elements" (1482 edition)

Diophantus (~200 CE) and The birth of literal algebra

Diophantus - (possibly ~250AD) - Alexandria

Arithmetica

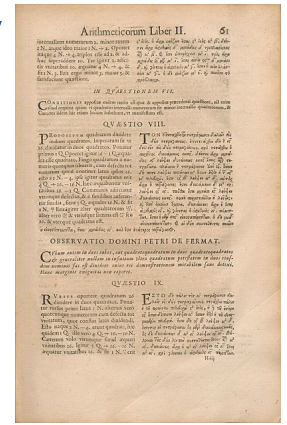
- Major advance in the solution of equation: Introduction to symbolism.
- Linear and quadratic equations
- Higher degree equations
- The method of false position

ALGEBRA

$$K^{\gamma} \alpha \zeta \gamma \Delta \Delta^{\gamma} \gamma \dot{M} \alpha$$

$$x^3 - 3x^2 + 3x - 1$$

Problem II.8 in the Arithmetica (edition of 1670), annotated with Fermat's comment which became Fermat's Last Theorem. Wikipedia



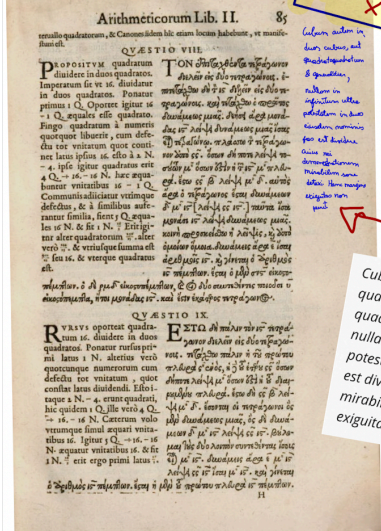
Fermat last theorem
 Is $n \geq 3$, there are no integers
 x, y and z such that
 $x^n + y^n = z^n$

Andrew Wiles presented a proof in 1993 - 1994

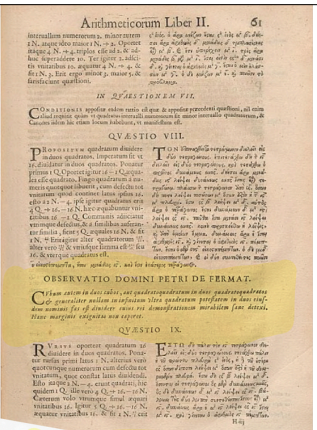


Fermat ~1600

Fermat last theorem
 Is $n \geq 3$, there are no integers
 x, y and z such that
 $x^n + y^n = z^n$

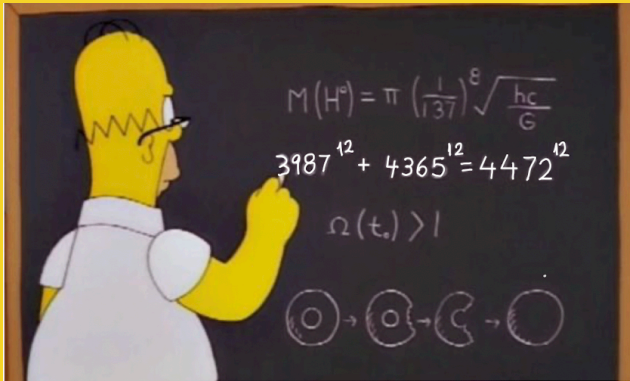


Cubum autem in duos cubos, aut quadratoquadratum in duos nullum in infinitum ultra quadratum potestatem in duos eiusdem nominis fas est dividere cuius rei demonstrationem exiguam non caperet.



It is impossible to separate a cube into two cubes, or a fourth power into two fourth powers, or in general, any power higher than the second, into two like powers. I have discovered a truly marvellous proof of this, which this margin is too narrow to contain.

Use a gadget (phone, computer..) that can only use numbers of 10 digits to compute first $3987^{12}+4365^{12}$ and second, 4472^{12} . Have we (jointly with Homer Simpson) disproved Fermat Last Theorem? Why or why not?



Solve the following problem by Diophantus. To split a given number (100) in two parts having a given difference (40).



Solve the following problem by Diophantus. To split a given number (100) in two parts having a given difference (40).

In fact, let the given arithmos be 100, and the excess M 40. To find the arithmoi. Let the smaller be assigned, v 1. Therefore, the larger will be v 1 M 40. Therefore, together they become v 2 M 40. But M 100 are given. Therefore, M 100 are equal to v 2 M 40. And from similars are similars. I subtract from 100, M 40 [and from the 2 arithmoi and of 40 units similarly 40 units]. Remainders v 2 equal M 60. Therefore each v becomes M 30. For the actualities. The smaller will be M 30 and the larger M 70, and the demonstration is obvious.



To split a given number (60) in two parts having a given ratio (3:1).



To split a given number (60) in two parts having a given ratio (3:1).

2. To divide the prescribed arithmos into two arithmoi in a ratio that's given.

In fact, let it be prescribed to divide 60 into two arithmoi in a ratio 3-times. Let the smaller be assigned ψ 1. Therefore, the larger will be ψ 3, and the larger is triple the smaller. It is required the two be equal to \aleph 60. But the two added are ψ 4. Therefore, ψ 4 are equal to \aleph 60. Therefore, ψ is \aleph 15. Therefore, the smaller will be \aleph 15 and the larger \aleph 45.



To find two numbers in a given ratio and such that their difference is also given.

Given ratio 5 : 1, given difference 20.

- Illustration by Rubens for "Opticorum libri sex philosophis juxta ac mathematicis utiles", by François d'Aguilon. It demonstrates the principle of a general perspective projection, of which the stereographic projection is a special case. Scan of a work published originally in 1613

Peter Paul Rubens - Opticorum libri sex philosophis juxta ac mathematicis utiles

