HOMEWORK 4

(1) Compute cohomology over $\mathbb{Z}$ of $\mathbb{RP}^2$ and the 2-torus.

(2) Compute cohomology over $\mathbb{Z}$ of the connect sum $M \# N$ of two 2-manifolds in terms of $H^*(M; \mathbb{Z})$ and $H^*(N; \mathbb{Z})$. Hence compute the cohomology of any (possibly unoriented) closed 2-manifold by using (1) and the classification of closed surfaces.

(3) Compute the groups $H_*(L(p, q); G)$ and $H^*(L(p, q); G)$ where $G = \mathbb{Q}$, $\mathbb{Z}/m\mathbb{Z}$, $m \in \mathbb{Z}$ and $\mathbb{Z}$ where $L(p, q)$ is the Lens space with p, q are coprime integers.

(4) Compute the cup product structure on any genus g surface using Δ-complex cohomology. Use this to show that any continuous map between a genus g surface and a genus h surface induces a trivial map on second cohomology if $h > g$.