# Homework 3

(1) Compute the cup product structure on any genus $g$ surface using $\Delta$-complex cohomology. Use this to show that any continuous map between a genus $g$ surface and a genus $h$ surface induces a trivial map on second cohomology if $h > g$.

(2) Compute cohomology over $\mathbb{Z}$ of the connect sum $M\#N$ of two 2-manifolds in terms of $H^*(M;\mathbb{Z})$ and $H^*(N;\mathbb{Z})$. Hence compute the cohomology of any (possibly unoriented) closed 2-manifold by using (1) and the classification of closed surfaces.

(3) Compute the groups $H_*(L(p,q);G)$ and $H^*(L(p,q);G)$ where $G = \mathbb{Q}$, $\mathbb{Z}/m\mathbb{Z}$, $m \in \mathbb{Z}$ and $\mathbb{Z}$ where $L(p,q)$ is the Lens space with $p, q$ are coprime integers.

(4) Compute $\widetilde{K}^*(\Sigma)$ as a group for any closed oriented surface $\Sigma$.

(5) Show that there is an isomorphism of rings:

$$\mathbb{Z}[H]/(H^{n+1}) \xrightarrow{\cong} K^*(\mathbb{CP}^n) \tag{1}$$

where $H$ is the tautological line bundle on $\mathbb{C}P^n$. Use the cellular structure and the long exact sequence prove this.

(6) Compute the Chern character map:

$$K^*(\mathbb{CP}^n) \cong \mathbb{Z}[H]/(H^{n+1}) \longrightarrow H^*(\mathbb{CP}^n;\mathbb{Q}). \tag{2}$$