Final exam practice problems
MAT 127
Dec 7, 2015

Name:		**ID #**:
(please print)		
Your section:	(see list below)	

This is a collection of practice problems for the final exam. Note that it is much longer than the actual exam.

No books or calculators. You are allowed to bring one letter-size sheet of paper with your notes.

You must show your reasoning, not just the answer. Answers without justification will get only partial credit.

Please cross out anything that is not part of your solution — e.g., some preliminary computations that you didn't need.

It is OK to leave the answer as an expression that involves some algebraic, trigonometric, exponential, or logarithmic functions (such as $e^3/\sqrt{2}$) — you do not have to convert it to a decimal value (which would be hard without a calculator). However, if the answer can be obviously simplified, please do so (e.g., do not leave $\sin(\pi)$ in the final answer)

Lecture 01	MWF 10:00 AM – 10:53 AM	Alexander Kirillov
Lecture 02	MW 5:30 PM – 6:50 PM	Mark McLean
Lecture 04	TUTH 5:30 PM – 6:50 PM	Sabyasachi Mukherjee

1. For each of the following sequence, determine whether it converges. If it converges, find the limit.

(a) $a_n = \frac{n^3+2}{2n^4+1}$

(b) $a_n = n\sin(\pi/n)$

(c) $a_n = \sqrt{2n+1} - \sqrt{2n-1}$

(d) $a_n = \frac{2^n}{n!}$

2. Let the sequence a_n be defined by $a_0 = \pi/4$, $a_{n+1} = \sin(a_n)$. Prove that this sequence has a limit and find this limit.

3. For each of the series below, determine whether the series converges.

(a)
$$\sum_{n=1}^{\infty} \frac{n^2-1}{n^2+4}$$

(b)
$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{2n+3}}$$

(c)
$$\sum_{n=0}^{\infty} \frac{n^2+4n}{n^3+1}$$

(d)
$$\sum_{n=0}^{\infty} \frac{ne^{2n}}{(2n)!}$$

4. (a) Find the radius of convergence of the following power series.

$$f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{(2n+1)x^n}{3^n}$$

(b) Write the indefinite integral

$$F(x) = \int f(x)\,dx$$

as a power series (inside the interval of convergence of the power series for $f(x)$.)

(c) The product $\cos(x)f(x)$ can also be written by a power series. Write the first three terms of this power series, up to x^2 term.

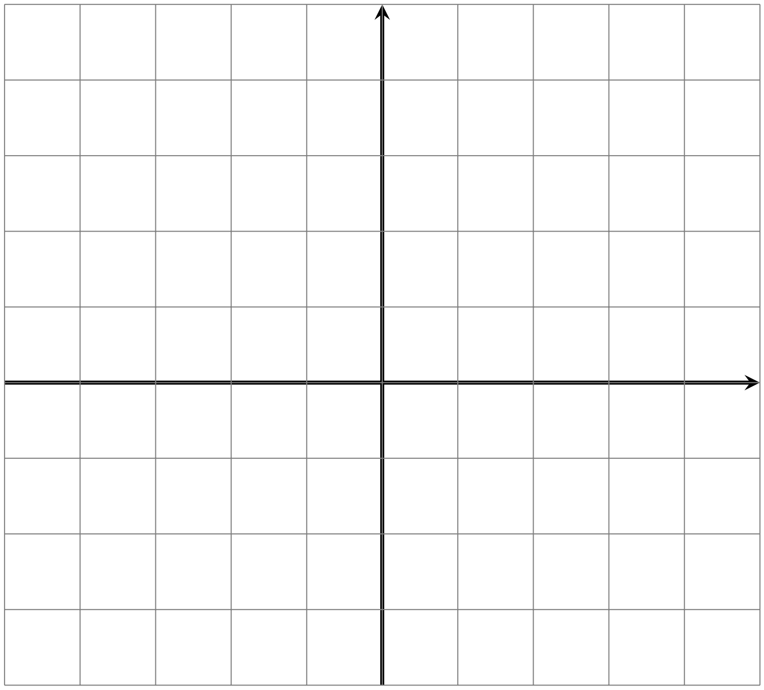
5. Let $f(x) = \tan^{-1}(2x) - 2x$.

(a) Write the Taylor polynomial $T_5(x)$ of degree 5 for $f(x)$, centered at $x = 0$.

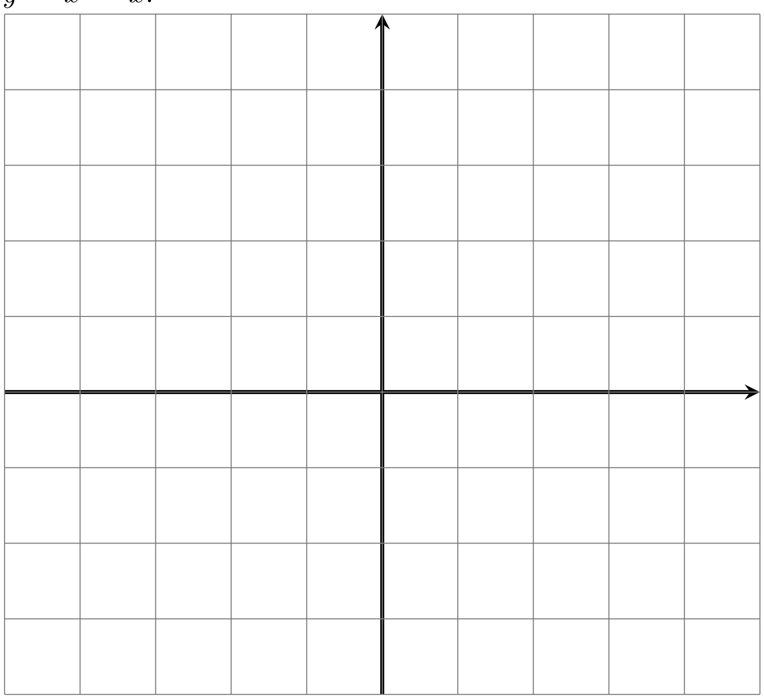
(b) Use the Taylor polynomial found in the previous part to give an approximate value of $f(0.1)$.

6. Draw direction fields for the following differential equations.

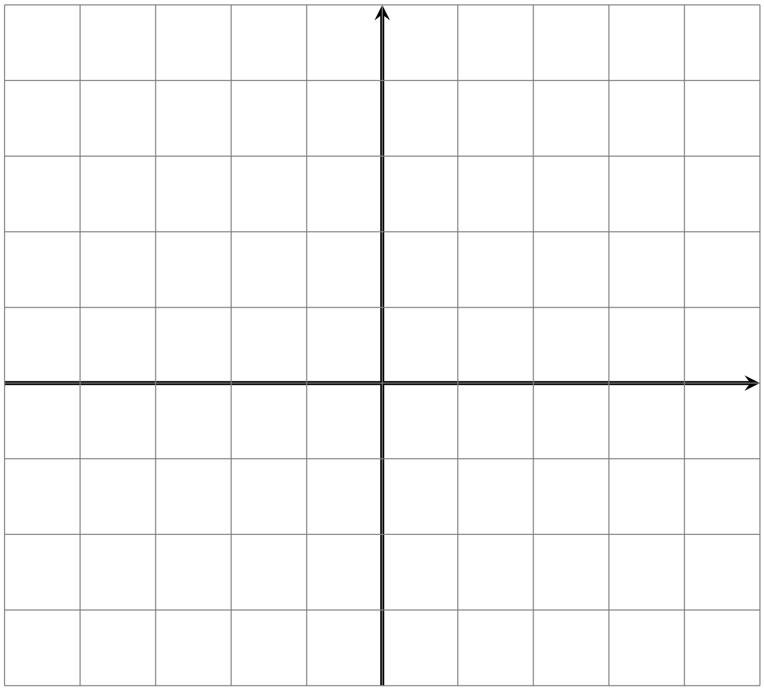
(a) $y' = y^2 - 1$



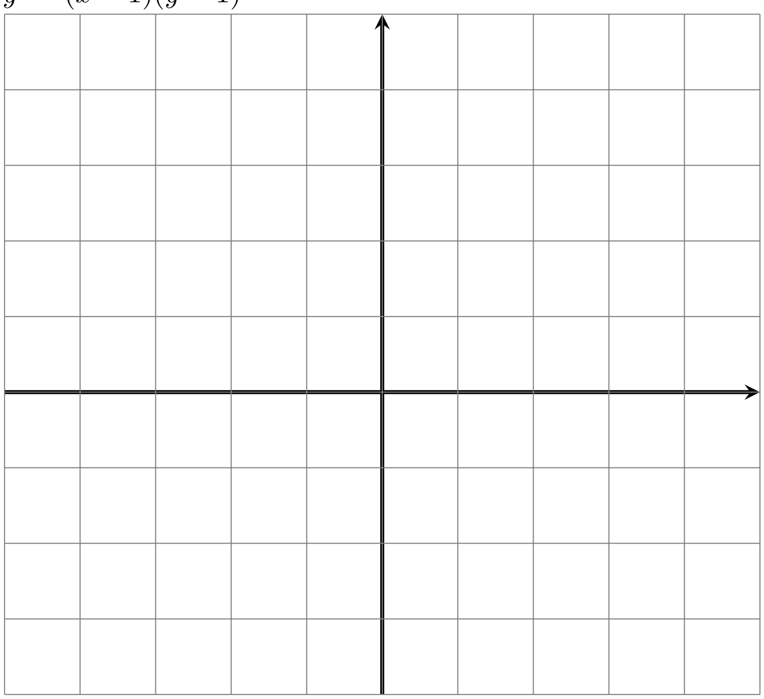
(b) $y' = x^2 - x$.



(c) $y' = x + y$.



(d) $y' = (x-1)(y-1)$



7. Use Eulers Method with step size 0.01 to estimate $y(0.04)$ where y satisfies: $y' = 3y, \quad y(0) = 2$.

8. Find the orthogonal trajectories of the family of curves $y^2 = kx^3$.

9. Solve the following initial value problems:

(a) $\frac{dy}{dx} = x^2(y^2 + 2y - 3),\ y(0) = 2$.

(b) $y' = y^2 - 1, \quad y(0) = 2$

(c) $y' = \frac{1}{y(\sqrt{1-x^2})}, y(0) = 1$.

10. A tank contains 100 L of pure water. Brine that contains 0.1 kg of salt per liter enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt is in the tank after 6 minutes?

11. A sample of tritium-3 decayed to 94.5% of its original amount after a year.
(a) What is the half-life of Tritium-3?
(b) How long would it take the sample to decay to 20% of its original amount?

12. A population of bees in a particular region satisfies the logistic equation with carrying capacity 10000. Suppose that there are only 1000 bees initially and 2000 bees after 2 years. How many bees are there after 3 years?

13. Find all equilibrium solutions of the following system of differential equations:

$$\frac{dW}{dt} = R^2 + RW$$
$$\frac{dR}{dt} = W^2 - R.$$