MAT131 Fall 2007 Final Review Sheet

The final exam will be **cumulative**. Please look at the review sheets for Midterms 1 and 2 to review the material from earlier in the semester. (We may choose to write up a few more problems from earlier in the semester. You can find many more such problems in the textbook and on exams from previous semesters.)

Particular emphasis on the final will be placed on the material which has not been tested on Midterms 1 or 2. Among this new material is the following.

(i) Sketching the graph of a function, including all relevant features. Among these features: is the function odd, even or neither, what are the vertical and horizontal asymptotes (if any), what are the discontinuities (if any), what are the local maxima and minima (if any), where is the function increasing and where decreasing, what are the inflection points (if any), where is the function concave up and where concave down.

(ii) L'Hôpital's rule. Recognize indeterminate forms. Simplify limits leading to $0/0$ and ∞/∞ indeterminate forms using l'Hôpital's rule. Know how to transform other indeterminate forms into one of these two types.

(iii) Optimization problems. Given a word problem attempting to maximize or minimize some quantity given a collection of constraints, turn this into a calculus problem for finding an absolute maximum or absolute minimum. Solve this calculus problem.

(iv) Newton's method. Understand the principle behind Newton's method and be able to carry it out with the aid of a calculator. Understand the pitfalls of Newton's method.

(v) Antiderivatives. Recognize the most common antiderivatives: those arising as the derivatives of x^n, trigonometric functions, exponential functions, logarithmic functions and inverse trigonometric functions.

(vi) Know how to set up a Riemann sum associated to a given integrand and a given interval. Be able to evaluate the limit of Riemann sums to compute the Riemann integral in the case of some simple integrands.

(vii) Know the statement of the Fundamental Theorem of Calculus. Understand how to use this to evaluate definite integrals when you can find a simple form for the antiderivative. Understand how the fundamental theorem always gives an antiderivative of a continuous function, where the antiderivative is defined in terms of the Riemann integral/definite integral.

(viii) Given a limit of sums, recognize when this is a limit of Riemann sums. Be able to use the fundamental theorem of calculus to evaluate this limit of Riemann sums.

(ix) Simplify antiderivatives using direct substitution.

(x) Evaluate definite integrals using direct substitution and the fundamental theorem of calculus.

Following are some practice problems. More practice problems are in the textbook as well as on the practice midterm.

Problem 1. In each of the following cases, sketch the graph of the given function. On your graph state whether the function is even, odd or neither. State whether or not the function is periodic, and state the period if it is periodic. Label all discontinuities and the type. Label all vertical and horizontal asymptotes. Label all local maxima and minima and state where the function is increasing and where decreasing. Label all inflection points and state where the function is concave up and where concave down.

(a)

$$y = x^2 - 4.$$

(b)

$$y = \frac{1}{x^2 - 4}$$

(c)
$$y = \frac{1}{x+1} - 2 + \frac{1}{x-1}.$$

(d)
$$y = \sin(x)$$

(e)
$$y = \tan(x)$$

(f)
$$y = \sin(x)\cos(x)$$

(g)
$$y = e^{-x^2/2}$$

(h)
$$y = \ln((x+1)^2/(x-1)^2).$$

(i)
$$y = \frac{e^x}{1+e^x}.$$

(j)
$$y = e^{-1/x^2}.$$

Problem 2 Let S be the square in the xy-plane centered at the origin, of edge length $\sqrt{2}$, and with diagonal edges of slopes $+1$ respectively -1. The equation for this square is
$$|x| + |y| = 1.$$

Let R be a square in the xy-plane centered at the origin whose horizontal and vertical edges are parallel to the x-axis and y-axis respectively. Among squares R which intersect S, what is the maximum possible area of the region lying outside the inner square and inside the outer square?

Problem 3 Compute the maximum volume of a right circular cone whose surface area (just of the cone, not of the "bottom" disk of the cone) is a fixed constant A. The surface area of a right circular cone is $A = \pi r s$ where r is the radius of the bottom disk and s is the *slant height*, i.e., the distance from the vertex of the cone to a point on the bottom circle of the cone. What is the ratio of radius to height for such a cone?

Problem 4 You will build a box in a corner of a room using the floor and two walls as sides of the box. To do this, remove a square from one corner of a square sheet of metal of edge length 10 feet. Fold the edges of the sheet meeting the missing square to form two sides of a box. The remaining square of the sheet forms the top of the box. Slide these three sides into the corner to form a box with square top. What is the maximum volume of this box?

Problem 5 Compute each of the following limits.

(a)

$$\lim_{x\to 0^+} \frac{1-\cos(x)}{\sin^2(x)}$$

(b)

$$\lim_{x\to 0^+} \ln(1-\cos(x)) - 2\ln(\sin(x))$$

(c)

$$\lim_{x\to\infty} \frac{1+x}{x^2}$$

(d)

$$\lim_{x\to\pi/2} \tan(x) - \sec(x)$$

(e)

$$\lim_{x\to\infty} x\ln\left(\frac{x}{x+1}\right).$$

Problem 6 By thinking about areas compute an antiderivative of $\sqrt{1-x^2}$. (Hint: Sketch the region whose area is the definite integral of this function from 0 to x.)

Problem 7 Consider the Riemann integral

$$\int_1^4 (2x+1)dx.$$

Partition the interval into n subintervals of equal length. Compute the Riemann sum S_n for this partition using right endpoints. Write down the value of this Riemann sum. Directly compute the limit as n goes to infinity to find the Riemann integral. Double-check your answer against the Fundamental Theorem of Calculus.

Problem 8 Consider the Riemann integral

$$\int_0^1 3^x dx.$$

Partition the interval into n subintervals of equal length. Compute the Riemann sum S_n for this partition using left endpoints. Write down the value of this Riemann sum. Directly compute the limit as n goes to infinity to find the Riemann integral (you may use L'Hôpital's rule to evaluate the limit). Double-check your answer against the Fundamental Theorem of Calculus.

Problem 9 Compute each of the following definite and indefinite integrals.

(a)

$$\int x^{-1/2} dx$$

(b)

$$\int (x + \sin(x)) dx$$

(c)

$$\int \frac{1}{\sqrt{1-x^2}} dx$$

(d)

$$\int \sec(\theta)\tan(\theta) d\theta$$

(e)

$$\int_{-2}^{2} x^4 dx.$$

(f)

$$\int_{-\pi/2}^{\pi/2} \cos(2x) - \cos(x) dx$$

(g)

$$\int_0^{1/2} \frac{1}{\sqrt{1-x^2}}dx$$

(h)

$$\int_{\pi/4}^{\pi/3} \sec^2(\theta)d\theta$$

Problem 10 Evaluate each of the following limits.

(a)

$$\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} 1.$$

(b)

$$\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n}(1-\frac{2i}{n})$$

(c)

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{n-2i}{n^2}.$$

(d)

$$\lim_{n\to\infty} \sum_{i=0}^{n-1} \frac{2^{i/n}}{n}$$

(e)

$$\lim_{m\to\infty} \sum_{i=1}^{m} \frac{m}{m^2+i^2}$$

Problem 11 Compute each of the following indefinite and definite integrals.

(a)

$$\int_a^b f'(cx)dx$$

(b)

$$\int x\sin(x^2)dx$$

(c)

$$\int_0^{\pi/4} \tan(x)dx$$

(d)

$$\int_0^{\pi/2} \sin^3(x)dx$$

(e)

$$\int \frac{\ln(x)}{x}dx$$

(f)

$$\int_1^2 \frac{e^{\ln(x)}}{x}dx$$

(g)

$$\int_0^{x^2} \frac{f'(\sqrt{t})}{\sqrt{t}}dt$$