Problem Set 3

Disclaimer For open-ended problems, part of the problem is to give a precise formulation. Especially for the problems in Part II, you should do as much of the problems as is useful to you. For each problem, it is important you understand how to verify all details. However, if you are pressed for time, you may write-up only the most important steps, instead of every detail.
Late homework policy. Late work will be accepted only with a medical note or for another approved reason.

Cooperation policy. You are strongly encouraged to work with others, but the final write-up must be entirely your own and based on your own understanding.

Part I. These problems are from the textbook. You are expected to read *all* the problems from the sections of the textbook covered that week. You are asked to write-up and turn-in only the problems assigned below.

Part II. These problems are not necessarily from the textbook. Often they will be exercises in commutative algebra, category theory, homological algebra or sheaf theory.

Part I(25 points)

(a)	(5 points)	p. 80,	Section II.2,	Problem 2.8
(b)	(5 points)	p. 80,	Section II.2,	Problem 2.13 (b) and (c)
(c)	(10 points)	p. 81,	Section II.2,	Problem 2.16
(d)	(5 points)	p. 81,	Section II.2,	Problem 2.17

Part II(25 points)

Problem 1(10 points) Let $\phi : A \to B$ be a ring homomorphism. Characterize when $\phi^* : \text{Spec } B \to \text{Spec } A$ is an isomorphism from Spec B to an open subset of Spec A with its induced scheme structure. For every example, is it true that B is isomorphic to $A[1/a]$ for some element a of A?

Problem 2(10 points) Work through Exercise II.2.18 on p. 81 of the textbook. Please note theproblem continues onto p. 82.

Problem 3(5 points) Describe, in a sense you make precise, the points, open sets and residue fields of the scheme Spec $\mathbb{Z}[x]$. Please see Exercise II.2.7 on p. 80 for the definition of *residue field*.

Extra credit(5 points) Give an example of a scheme such that no singleton set is a closed subset.
Hint: Such a scheme is necessarily not quasi-compact.