MAT 203 FINAL EXAM

WEDNESDAY DECEMBER 10, 2025
2:15–5:00PM

Name: ______________________________ ID: ______________

Instructions.

(1) Fill in your name and Stony Brook ID number.

(2) This exam is closed-book; no electronic devices. You are only allowed to have one (1) sheet of your own notes.

(3) You have 2 hours and 30 minutes to complete this exam.

(4) You must justify all your answers and show all your work. Even a correct answer without any justification will result in no credit.

1. (a) (5 pts) Compute $(\vec{u} + \vec{v}) \cdot (\vec{u} \times \vec{v})$, where $\vec{u} = \langle 0, 1, 2 \rangle$ and $\vec{v} = \langle -2, 1, 1 \rangle$.

(b) (5 pts) Consider the vector $\vec{w} = \langle a, b, c \rangle$ and compute $\vec{w} \times \vec{w}$.

2. In each of the following problems, either compute the limit, or show that it does not exist.

(a) (5 pts) $\lim_{(x,y)\to(0,0)} \frac{x+y}{x^2-y^2}$

(b) (5 pts) $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x+y}$

3. Consider the function $f(x, y, z) = z + e^{2x-y}$.

(a) (2 pts) Compute the gradient $\nabla f(x, y, z)$.

(b) (3 pts) Compute the divergence $\nabla \cdot \nabla f(x, y, z)$.

(c) (5 pts) Compute the curl $\nabla \times \nabla f(x, y, z)$.

4. Consider the surface S in space given by the equation $z = x^2 + xy + y^2$.

(a) (7 pts) Find the equation for the plane that is tangent to the surface S at the point $(x, y, z) = (2, -1, 3)$.

(b) (3 pts) Find a unit normal vector to the surface S at the point $(2, -1, 3)$.

5. Consider the vector field $\vec{F}(x,y) = \langle 3x^2 + 6xy, 3x^2 + 6y \rangle$.

(a) (7 pts) Prove that $\vec{F}$ is conservative by finding a potential function ϕ.

(b) (3 pts) Compute the line integral $\int_C \vec{F} \cdot d\vec{r}$, where C is the graph of the function $f(x) = x^3$ for $-1 \le x \le 1$, by using your result in part (a).

6. (10 pts) Consider the region R in space determined by the inequalities $0 \leq z \leq x + y$, $0 \leq y \leq x$, and $1 \leq x \leq 2$. Compute the triple integral

$$\iiint_R 2e^z \, dxdydz.$$

7. (10 pts) Consider the vector field $\vec{F}(x, y) = \langle e^x, y + x^2 + xy \rangle$, and let C be the curve in the plane consisting of the upper half of the unit circle and the x-axis, oriented counterclockwise. Compute the line integral

$$\int_C \vec{F} \cdot d\vec{r}.$$

8. (10 pts) Find the global minimum and global maximum of the function $f(x, y) = \frac{x^3}{3} + y^2$ on the unit disk in the xy-plane.

9. (10 pts) Consider the surface P parametrized by $\vec{r}(u,v) = \langle u, v, -u+v-1 \rangle$ for $0 \leq u \leq 2$ and $0 \leq v \leq 1$. Consider the vector field $\vec{F}(x,y,z) = \langle 2x-y, z, y \rangle$ and compute the surface integral $\iint_P \vec{F} \cdot d\vec{r}$.