MAT 203 L1 Mon Jan 26

---

§ Admin stuff: (see syllabus)

Me: Johan Asplund Math 3-116

Office hours: MW 4–5 pm

Course web page: Tue 5-6 pm (MLC)

math.stonybrook.edu/~jasplund/mat203_Spr26

Textbook: Openstax Calculus Volume 3
(free; available online)

Weekly homeworks/quizzes: $\boxed{\begin{array}{l}2.2.88\\2.4.190\end{array}}$

- Weekly pool of 4–6 problems.

- 2 are homework problems to be handed in at recitation (due next week)
- Quiz last 15 min of each recitation where you will be asked to solve 1 (randomly chosen) problem from the pool.

$\boxed{\begin{array}{l}2.1.6\\2.1.10\\2.2.92\\2.3.126\\2.4.186\end{array}}$

First quiz: Next week.

Important dates:

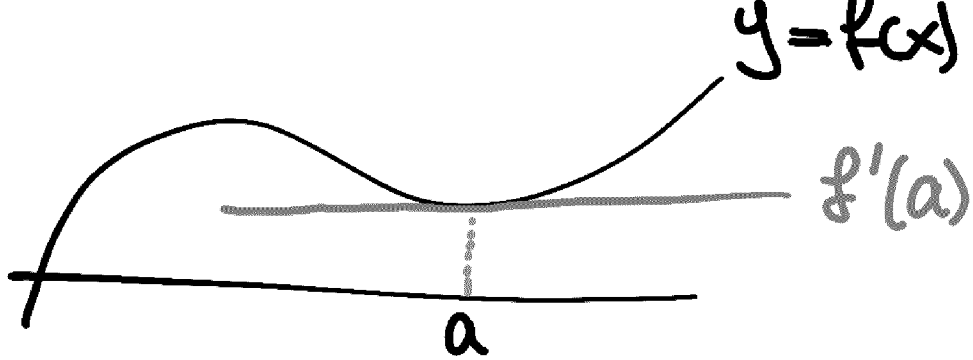
- Midterm 1: Mon Feb 23
- Midterm 2: Mon Apr 13
- Final exam: W May 13

§ Ideas/goals of MAT 203

Calculus I:

$$f(x) = x^3 - x$$

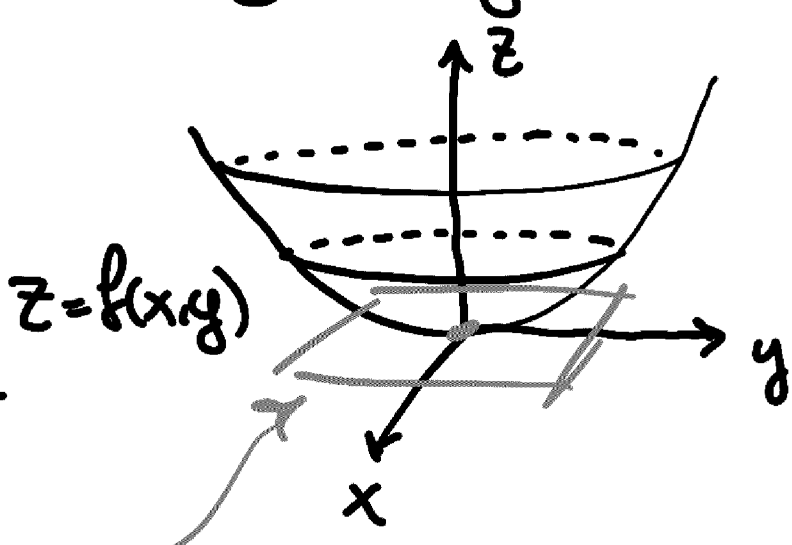
$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$



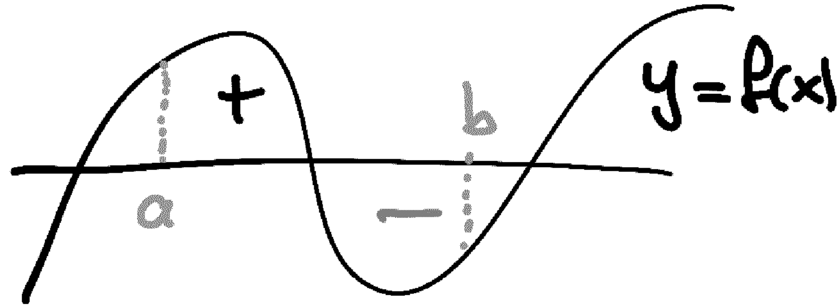
What if $f(x,y) = x^3 - x + y^2 - y$? What is the rate of change as x changes? as y changes? or both?

$$f(x,y) = x^2 + y^2$$

We do not have a tangent line, but a tangent plane.



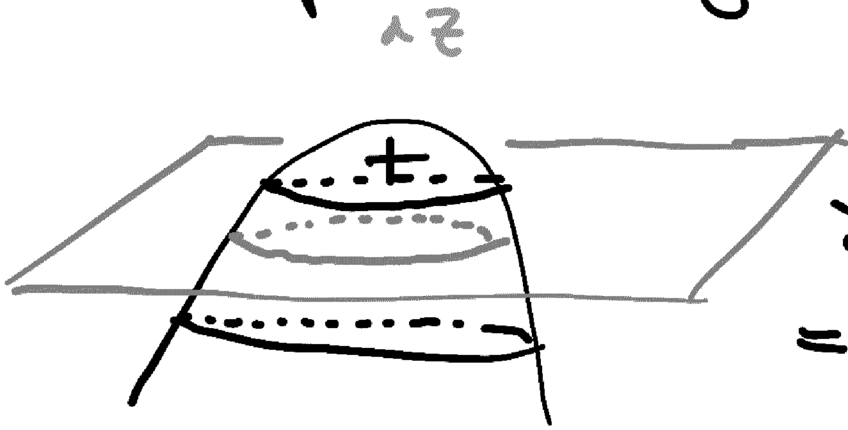
Calculus II:



$$\int_a^b f(x)\,dx = \text{signed area under the graph } y=f(x).$$

$=$ Summing up valus of $f(x)$ over the interval $[a,b]$.

For fenctions of multiple variables one computes a signul volume:



$f(x,y) = 1-x^2-y^2$.

$$V = \iint_{x^2+y^2\le 1} f(x,y)\,dx\,dy$$

There are at least 4 other kinds of integrals we will learn in this class.

Fund thm of calculus:

$$F(x) = \int_a^x f(t)\,dt,$$ then $F(x)$ is differentiable & $F'(x) = f(x)$.

There are different versions of this for functions of multiple variables:

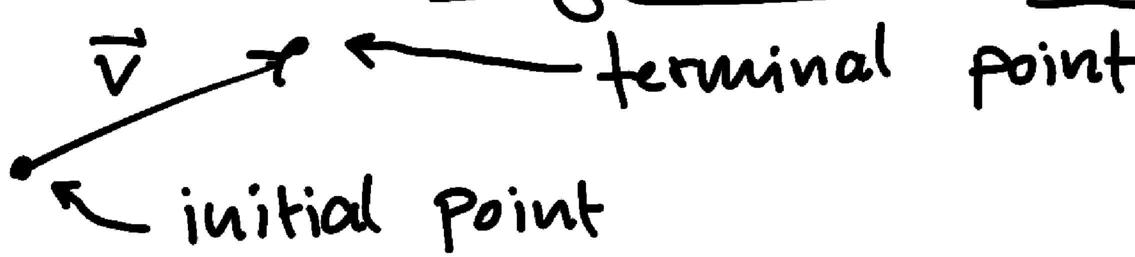
- Green's theorem
- Stokes' theorem
- Gauss' theorem (divergence theorem).

- Important in applications such as physics (e.g. Maxwell's equations).

---

§2.1, 2.2 Vectors in the plane & space

Def: A vector is a quantity that has both a magnitude & direction.

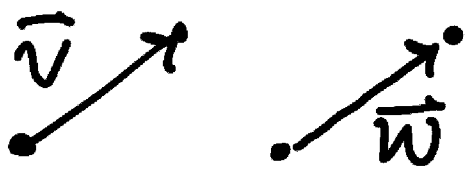


We write an arrow over v to indicate that v is a vector.
In print, boldface fonts are sometimes used, but it's hard to use in handwriting.
Magnitude of $\vec{v}$ is denoted by $\|\vec{v}\|$.

- A vector from the point P to the point Q is denoted $\overrightarrow{PQ}$.

$$Q \bullet \xleftarrow{\ \overrightarrow{PQ}\ } \bullet\ P$$

- Two vectors are "equivalent" (or the same) if they have the same magnitude & the same direction.



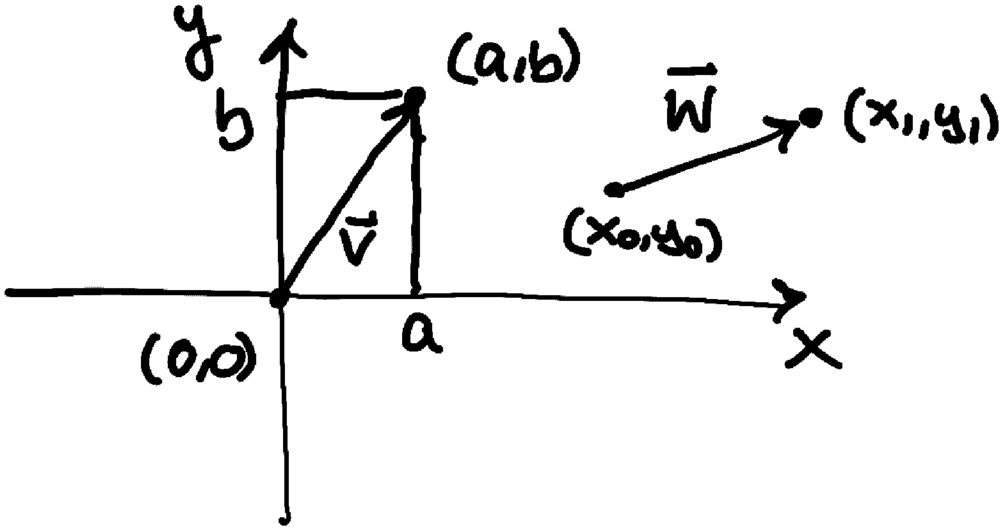
$\vec{v} = \vec{w}$.

Even if their initial and terminal points are <u>different</u>, their magnitudes & directions are the same.

A vector $\vec{v}$ with $\|\vec{v}\| = 0$ is called the zero vector. It has no direction.
(It's the only vector without one.)

Cartesian plane:

$\mathbb{R}^2 = \{(x,y) \mid x,y \in \mathbb{R}\}$



$\vec{v}$ starts at $(0,0)$ & ends at (a,b).
it's denoted by $\vec{v} = \langle a,b \rangle$, where a and b are the components of $\vec{v}$.

The vector $\vec{w}$ is equivalent to the vector starting at $(0,0)$ and ending at (x_1-x_0, y_1-y_0), so
$\vec{w} = \langle x_1-x_0, y_1-y_0 \rangle$.

Zero vector: $\vec{0} = \langle 0,0 \rangle$.

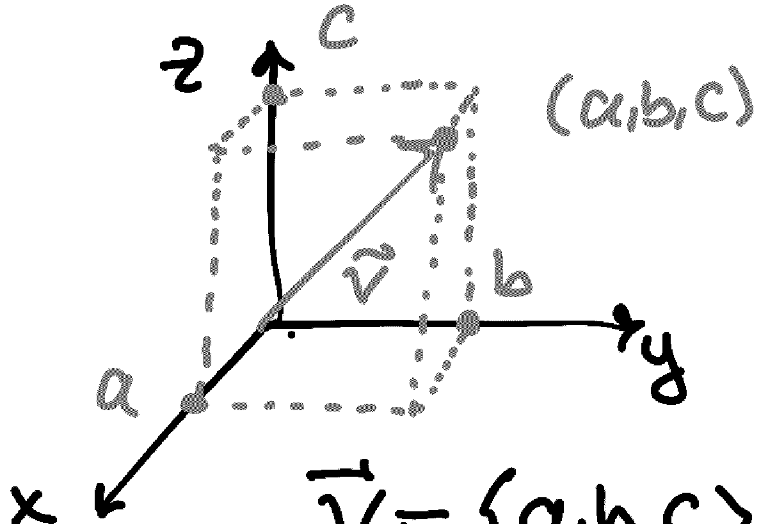
Magnitude of $\langle a,b \rangle$ is
$\|\langle a,b \rangle\| = \sqrt{a^2+b^2}$

Magnitude of vector from (x_0, y_0) to (x_1, y_1) is

$$\|\langle x_1 - x_0, y_1 - y_0 \rangle\| = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}$$

Cartesian space:

$$\mathbb{R}^3 = \{(x, y, z) \mid x, y, z \in \mathbb{R}\}.$$



$$\vec{v} = \langle a, b, c \rangle$$

$\vec{v}$ starts at $(0,0,0)$ and ends at (a,b,c).

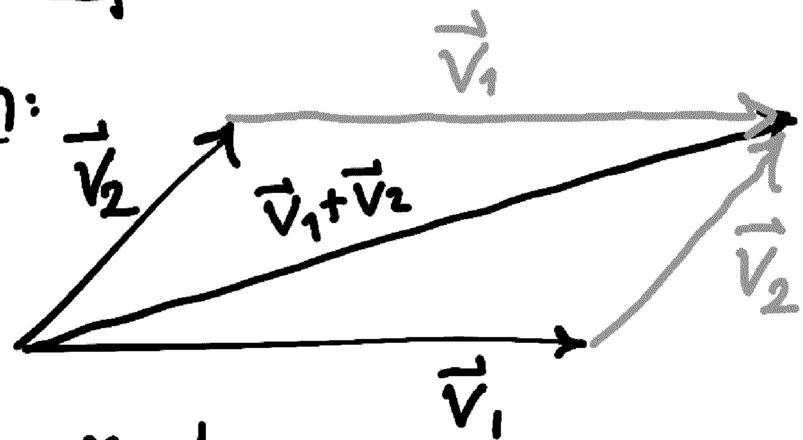
$$\|\vec{v}\| = \sqrt{a^2 + b^2 + c^2}.$$

If $\vec{w}$ starts at (x_0, y_0, z_0) and ends at (x_1, y_1, z_1), then

$$\|\vec{w}\| = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}.$$

Vector operations:

Addition:



In components:

$\vec{v}_1=\langle x_1,y_1,z_1\rangle$, $\vec{v}_2=\langle x_2,y_2,z_2\rangle$ then

$\vec{v}_1+\vec{v}_2=\langle x_1+x_2, y_1+y_2, z_1+z_2\rangle$

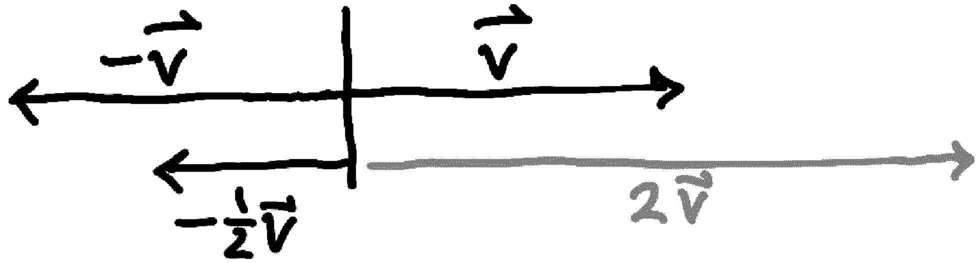
note: $\|\vec{v}_1+\vec{v}_2\| \leq \|\vec{v}_1\| + \|\vec{v}_2\|$.

(triangle inequality)

Scalar multiplication:

If $\vec{v}$ is a vector, and r is a scalar (real number)

The vector $r\vec{v}$ is the vector in the same direction (if $r>0$), or opposite direction (if $r<0$) as $\vec{v}$, but w/ length $|r|\,\|\vec{v}\|$.



If $\vec{v} = \langle x, y, z \rangle$, then
$r\vec{v} = \langle rx, ry, rz \rangle$. Note $0 \cdot \vec{v} = \vec{0}$
for any vector $\vec{v}$.

- Now we can subtract vectors:
$\vec{v}_1 - \vec{v}_2 = \vec{v}_1 + (-1)\vec{v}_2$.
$= \langle x_1, y_1, z_1 \rangle + (-1)\langle x_2, y_2, z_2 \rangle = \langle x_1 - x_2, y_1 - y_2, z_1 - z_2 \rangle$

- A <u>unit vector</u> is a vector of length 1. Given a vector $\vec{v} \neq \vec{0}$, its length is $\|\vec{v}\|$. Therefore
$\frac{1}{\|\vec{v}\|}\vec{v}$ has the same direction as $\vec{v}$, but $\left\|\frac{1}{\|\vec{v}\|}\vec{v}\right\| = \frac{1}{\|\vec{v}\|} \cdot \|\vec{v}\| = 1$
So $\frac{1}{\|\vec{v}\|}\vec{v}$ is a unit vector.
$\vec{v} = \langle x, y, z \rangle$, $\|\vec{v}\| = \sqrt{x^2 + y^2 + z^2}$, then

$$\frac{1}{\|\vec{v}\|}\vec{v} = \left\langle \frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}} \right\rangle$$

Properties of vector operations:

$\vec{u}, \vec{v}, \vec{w}$ vectors, r, s scalars

(1) $\vec{u}+\vec{v} = \vec{v}+\vec{u}$

(2) $(\vec{u}+\vec{v})+\vec{w} = \vec{u}+(\vec{v}+\vec{w})$

(3) $\vec{u}+\vec{0} = \vec{u}$

(4) $\vec{u}+(-\vec{u}) = \vec{0}$

(5) $r(s\vec{u}) = (rs)\vec{u}$

(6) $(r+s)\vec{u} = r\vec{u}+s\vec{u}$

(7) $r(\vec{u}+\vec{v}) = r\vec{u}+r\vec{v}$

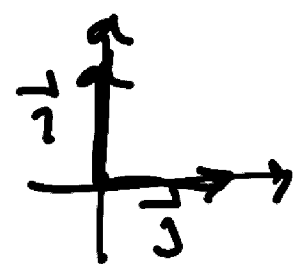
(8) $1\vec{u} = \vec{u}$, $0\vec{u} = \vec{0}$

(9) $\|\vec{u}+\vec{v}\| \leq \|\vec{u}\|+\|\vec{v}\|$ (triangle inequality)

Standard unit vectors

Plane: $\vec{i} = \langle 1, 0 \rangle$

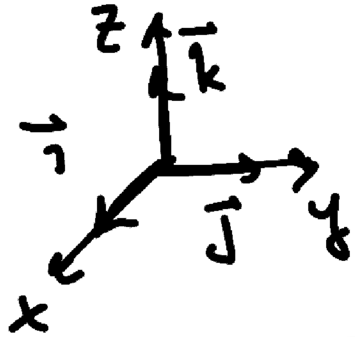
$\vec{j} = \langle 0, 1 \rangle$



$\vec{v} = \langle x, y \rangle = x\vec{i} + y\vec{j}$

<u>Space</u>: $\vec{i} = \langle 1,0,0\rangle$, $\vec{j} = \langle 0,1,0\rangle$
$\vec{k} = \langle 0,0,1\rangle$

$\vec{v} = \langle x,y,z\rangle = x\vec{i} + y\vec{j} + z\vec{k}$.



<u>Ex</u>. $P=(0,0,1)$, $Q=(1,0,1)$, $R=(1,2,3)$
$S=(0,1,0)$.

Let $\vec{u} = \overrightarrow{PQ}$, $\vec{v} = \overrightarrow{RS}$

$\vec{u} = \langle 1-0, 0-0, 1-1\rangle = \langle 1,0,0\rangle$

$\vec{v} = \langle 0-1, 1-2, 0-3\rangle = \langle -1,-1,-3\rangle$

$\vec{u}+\vec{v} = \langle 1-1, 0-1, 0-3\rangle = \langle 0,-1,-3\rangle$

$\|\vec{u}+\vec{v}\| = \sqrt{0^2+(-1)^2+(-3)^2} = \sqrt{1+9} = \sqrt{10}$.
