MAT 203 MIDTERM I

THURSDAY SEPTEMBER 25, 2025
11:00–12:20PM

Name: ______________________________ ID: ____________

Instructions.

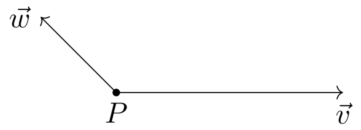
(1) Fill in your name and Stony Brook ID number.

(2) This exam is closed-book and closed-notes; no electronic devices.

(3) You have 80 minutes to complete this exam.

(4) You must justify all your answers and show all your work. Even a correct answer without any justification will result in no credit.

1. (a) (5 pts) In the diagram below, draw the vectors $-\vec{v}$ and $\vec{v}+\vec{w}$, such that both of their initial points are at P.



(b) (5 pts) Consider the vectors $\vec{u} = \langle 0, 1, -2\rangle$, $\vec{v} = \langle -2, 1, 1\rangle$, and $\vec{w} = \langle 0, 0, 1\rangle$. Compute the following: $\vec{v} \cdot (\vec{u} + \vec{w}) - \vec{w} \cdot (\vec{v} + \vec{u})$.

Solution. Note first $\vec{v}\cdot(\vec{u}+\vec{w})-\vec{w}\cdot(\vec{v}+\vec{u}) = \vec{v}\cdot\vec{u}+\vec{v}\cdot\vec{w}-\vec{w}\cdot\vec{v}-\vec{w}\cdot\vec{u} = \vec{v}\cdot\vec{u}-\vec{w}\cdot\vec{u}$. Then we compute

$$\vec{v}\cdot\vec{u} = \langle 0,1,-2\rangle\cdot\langle -2,1,1\rangle = -1$$
$$\vec{w}\cdot\vec{u} = \langle 0,0,1\rangle\cdot\langle 0,1,-2\rangle = -2$$

and therefore $\vec{v}\cdot(\vec{u}+\vec{w})-\vec{w}\cdot(\vec{v}+\vec{u}) = -1-(-2) = 1$. □

2. (a) (5 pts) Consider the line ℓ in the plane that passes through the point $(-1,-3)$, and has direction vector $\vec{v} = \langle 2, 1\rangle$. Find the equation of the line and write your answer on the form $y = kx + m$.

Solution. The slope is the ratio between the y- and the x-components of $\vec{v}$, so we find $k = \frac{1}{2}$. Therefore the linear form of the line is

$$y - (-3) = \frac{1}{2}(x - (-1)) \Longleftrightarrow y = \frac{1}{2}x + \frac{1}{2} - 3 = \frac{1}{2}x - \frac{5}{2}.$$

□

(b) (5 pts) Find a unit normal vector to the plane $2x - y + 4z + 1 = 0$.

Solution. A normal vector is given by $\vec{n} = \langle 2, -1, 4\rangle$. To find a unit normal vector we normalize it:

$$\frac{\vec{n}}{||\vec{n}||} = \frac{\langle 2, -1, 4\rangle}{\sqrt{2^2 + (-1)^2 + 4^2}} = \frac{\langle 2, -1, 4\rangle}{\sqrt{21}}.$$

□

3. (10 pts) Find the three radii of the ellipsoid described by the equation

$$10x^2 - 20x + 5y^2 + 10y + 2z^2 = -5.$$

Solution. We first complete the squares in the x and y variables and rewrite the left hand side as follows

$$\begin{aligned} 10x^2 - 20x + 5y^2 + 10y + 2z^2 &= 10(x^2 - 2x) + 5(y^2 + 2y) + 2z^2 \\ &= 10(x^2 - 2x + 1 - 1) + 5(y^2 + 2y + 1 - 1) + 2z^2 \\ &= 10(x-1)^2 - 10 + 5(y+1)^2 - 5 + 2z^2. \end{aligned}$$

Then we set this equal to -5 and find

$$\begin{aligned} 10(x-1)^2 - 10 + 5(y+1)^2 - 5 + 2z^2 = -5 &\iff 10(x-1)^2 + 5(y+1)^2 + 2z^2 = 10 \\ &\iff (x-1)^2 + \frac{(y+1)^2}{2} + \frac{z^2}{5} = 1. \end{aligned}$$

The three radii (in the x, y, and z-directions, respectively) are therefore 1, $\sqrt{2}$, and $\sqrt{5}$. □

4. Consider the curve S that is parametrized by $\vec{r}(t) = \langle \cos t, \sin t, t^2 \rangle$.

(a) (5 pts) Recall that the *principal unit tangent vector* of a curve with parametrization $\vec{r}(t)$ is given by $\vec{T}(t) = \vec{r}\,'(t)/||\vec{r}\,'(t)||$.
Find the principal unit tangent vector of S.

Solution. We compute the derivative to be $\vec{r}\,'(t) = \langle -\sin t, \cos t, 2t \rangle$, and its magnitude is

$$||\vec{r}\,'(t)|| = \sqrt{(-\sin t)^2 + (\cos t)^2 + (2t)^2} = \sqrt{4t^2+1}.$$

Therefore the principal tangent vector is

$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{||\vec{r}\,'(t)||} = \frac{\langle -\sin t, \cos t, 2t \rangle}{\sqrt{4t^2+1}}.$$

□

(b) (5 pts) Compute the integral $\int_0^3 ||\vec{r}\,'(t)||^2 \, dt$.

Solution. From part (a), we have $||\vec{r}\,'(t)||^2 = 4t^2+1$. Then, we compute the integral

$$\int_0^3 ||\vec{r}\,'(t)||^2 \, dt = \int_0^3 4t^2 + 1 \, dt = \left[\frac{4t^3}{3} + t + C \right]_0^2 = 4 \cdot \frac{3^3}{3} + 3 = 39.$$

□