

MAT 127

COVERAGE OF MIDTERM II

The midterm will consist of 5 problems, each one worth 20 points each. It will include problems on topics on **Taylor and Maclaurin series, slope fields and Euler's method, separable ODEs and modeling using ODEs**. Absolute and conditional convergence will *not* be covered on the second midterm (but it will be covered on the final).

Below is a more detailed list of learning goals for each section. Keep in mind that because the midterm only consists of 5 problems, it is impossible to test each and every one of the learning goals listed below. All section references are to Stewart.

1. LEARNING GOALS

Taylor and Maclaurin series: (All of §8.7.) Learning goals:

- State the general form of Taylor and Maclaurin series and polynomials.
- Find Taylor series and Taylor polynomials with a given center, by three methods: (1) computing derivatives and using the Taylor formula, (2) using substitution and arithmetic operations on known basic Maclaurin series such as $\sin x$, $\cos x$, e^x , $\frac{1}{1-x}$, $\log(1+x)$, or (3) using differentiation or integration of known series.
- **Note: You will *not* need to memorize common Taylor series: A reference page will be provided on the second midterm.**

Differential equations, slope fields and Euler's method: (All of §7.1 and §7.2.)

Learning goals:

- Explain what a differential equation is and find the order of the given differential equation.
- Understand the notion of a solution of a differential equation; check whether a given function is a solution to the given differential equation or to the initial value problem.
- Understand slope fields as a geometric interpretation of a first order ODE and use them to derive properties solutions (plot a solution curve for a given initial value problem; determine whether the solution to an initial value problem is increasing or decreasing; determine asymptotic behavior of a solution of the initial value problem).
- Being able to sketch a slope field.
- Understand and apply Euler's method to find approximations of solutions; provide a geometric interpretation of the method.

Separable ODEs and modeling: (All of §7.3 and §7.4.)

Learning goals:

- Identify separable ODEs; apply method of separation of variables to solve differential equations; solve initial-value problems.
- Compose differential equations and initial value problems to model processes described by word problems; solve the resulting problems and give a real-life interpretation of the solution; compare the behavior of solutions to qualitative predictions obtained by vector fields; verify whether the solutions make sense in the context of the problem
- Work with exponential models for growth and decay, recognize the corresponding differential equations.

2. REFERENCE PAGE THAT WILL BE PROVIDED AT THE MIDTERM

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \dots \quad |x| < 1$$

$$\log(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \quad |x| < 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \quad x \in \mathbb{R}$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \quad x \in \mathbb{R}$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \quad x \in \mathbb{R}$$

$$\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \quad |x| < 1$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = \binom{k}{0} + \binom{k}{1} x + \binom{k}{2} x^2 + \binom{k}{3} x^3 + \dots, \quad |x| < 1$$

$$\binom{k}{0} = 1, \quad \binom{k}{n} = \frac{k(k-1)\cdots(k-n+1)}{n!}$$

The **Taylor series of $f(x)$ centered at $x = a$** is given by

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!} (x-a)^2 + \frac{f'''(a)}{3!} (x-a)^3 + \dots$$