Print your name: ____________________

Answer each question completely. You must justify your answers to get credit. Even a correct answer with no justification will get no credits. Each problem is worth 5 points.

1. Write $\frac{3+\sqrt{2}i}{2+i}$ in the form $a+bi$.

Solution. Multiply with the complex conjugate to the denominator, which is $2-i$, to obtain

$$\frac{3+\sqrt{2}i}{2+i} = \frac{(3+\sqrt{2}i)(2-i)}{(2+i)(2-i)} = \frac{6+2\sqrt{2}i-3i-\sqrt{2}i^2}{4-i^2} = \frac{(6+\sqrt{2})+(2\sqrt{2}-3)i}{5}$$
$$= \frac{6+\sqrt{2}}{5} + \frac{2\sqrt{2}-3}{5}i$$

□

2. Write $(1+i)^8$ in the form $a+bi$.

Solution. First we write $1+i$ on polar form. We get $|1+i| = \sqrt{1^2+1^2} = \sqrt{2}$ and $\arg(1+i) = \arctan\left(\frac{1}{1}\right) = \arctan(1) = \frac{\pi}{4}$, and therefore

$$1+i = \sqrt{2}e^{i\frac{\pi}{4}}.$$

Therefore

$$(1+i) = \left(\sqrt{2}e^{i\frac{\pi}{4}}\right)^8 = (\sqrt{2})^8 e^{i\frac{8\pi}{4}} = 16e^{2\pi i} = 16.$$

□