

Print your name: _____

Answer each question completely. You must justify your answers to get credit. Even a correct answer with no justification will get no credits. The problem is worth 10 points.

1. Suppose that you have a supply of 200 mg of the radioactive isotope xenon-135. If $m(t)$ is the mass (in mg) of xenon-135 that remains after t hours, it satisfies the initial-value problem

$$\frac{dm}{dt} = \frac{3m}{100}, \quad m(0) = m_0,$$

where m_0 is the initial mass.

- (a) (5 pts) Solve the initial-value problem.

$$\frac{dm}{dt} = \frac{3m}{100} \Leftrightarrow \int \frac{1}{m} dm = \int \frac{3}{100} dt$$

$$\Leftrightarrow \log|m| = \frac{3t}{100} + C \Leftrightarrow m = e^{\frac{3t}{100} + C} = De^{\frac{3t}{100}}$$

$$m(0) = m_0 = 200 \Leftrightarrow De^0 = 200$$

$$\Leftrightarrow D = 200$$

Answer: $m(t) = 200e^{\frac{3t}{100}}$

- (b) (5 pts) When is the mass of your supply of xenon-135 equal to 100 mg?

want to solve $m(t) = 100$

$$\Leftrightarrow 100 = 200e^{\frac{3t}{100}}$$

$$\Leftrightarrow \frac{1}{2} = e^{\frac{3t}{100}}$$

$$\Leftrightarrow t = \frac{100}{3} \log\left(\frac{1}{2}\right) \approx -10 \text{ hours}$$

Note: I noticed that there's a typo in the problem statement. The ODE should be $\frac{dm}{dt} = -\frac{3m}{100}$ leading to the intended answer of approx 10 h.