Print your name: _____

Answer each question completely. You must justify your answers to get credit. Even a correct answer with no justification will get no credits. Each problem is worth 5 points.

For reference:
$$\sin x = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

1. Find the Maclaurin polynomial of degree 2 of $f(x) = x\sin x$

$$x\sin x = x\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$= \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+2}}{(2n+1)!} = \boxed{x^2} - \frac{x^4}{3!} + \cdots$$

$$\boxed{M_2(x) = x^2}$$

2. Compute the limit $\lim\limits_{x\to 0}\dfrac{\sin(2x) - x}{x^3}$

$$\frac{\sin(2x) - x}{x^3} = \frac{2x - \frac{(2x)^3}{3!} + \cdots - x}{x^3} = \frac{x - \frac{(2x)^3}{3!} + \cdots}{x^3}$$

$$= \underbrace{\frac{1}{x^2}}_{\to\infty} - \underbrace{\frac{8}{3!} + \cdots}_{\to -\frac{8}{6}} \longrightarrow \boxed{\infty} \text{ as } x\to 0.$$