Print your name: ______________________________

Answer each question completely. You must justify your answers to get credit. Even a correct answer with no justification will get no credits.

1. Let $\{a_n\}_{n=1}^{\infty}$ satisfy $a_1 = 1$ and $a_n = a_{n-1} - \frac{1}{n(n-1)}$ for $n \geq 2$. Write the first four terms of the sequence and then give a formula for a_n that does not depend explicitly on the previous terms.

$a_1 = 1$

$a_2 = a_1 - \frac{1}{2(2-1)} = 1 - \frac{1}{2} = \frac{1}{2}$

$a_3 = a_2 - \frac{1}{3\cdot(3-1)} = \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$

$a_4 = a_3 - \frac{1}{4\cdot(4-1)} = \frac{1}{3} - \frac{1}{12} = \frac{1}{4}$

$\boxed{a_n = \frac{1}{n}}$ Check that it satisfies the recursion:

$$a_{n-1} - \frac{1}{n(n-1)} = \frac{1}{n-1} - \frac{1}{n(n-1)} = \frac{n}{n(n-1)} - \frac{1}{n(n-1)} = \frac{1}{n} = a_n$$

2. Let $a_n = \ln(2n^2+1) - \ln(n^2+1)$. Does the sequence $\{a_n\}_{n=1}^{\infty}$ converge or diverge? If it converges, calculate the limit. If it diverges, explain why.

$$\lim_{n\to\infty} \ln(2n^2+1) - \ln(n^2+1)$$

$$= \lim_{n\to\infty} \ln\left(\frac{2n^2+1}{n^2+1}\right) = \left[\begin{array}{l}\ln \text{ is} \\ \text{continuous}\end{array}\right]$$

$$= \ln\left(\lim_{n\to\infty} \frac{2n^2+1}{n^2+1}\right) = \ln\left(\lim_{n\to\infty} \frac{2+\frac{1}{n^2}}{1+\frac{1}{n^2}}\right)$$

$$= \ln\left(\frac{2+0}{1+0}\right) = \ln(2).$$

The Sequence converges to $\ln(2)$