Print your name: ______________________________

Answer each question completely. You must justify your answers to get credit. Even a correct answer with no justification will get no credits.

Consider the series $\sum_{n=1}^{\infty} \frac{1}{2n^3 - 1}$.

1. Thinking about what you have learned so far, what do you **think** the behavior of this series is? No justification needed. (Answer this question before the next questions; you are **not** allowed to change your answer.) (1 pts)

■ Convergent ☐ Divergent

2. Apply the **divergence test** to check for convergence or divergence. What is the conclusion? (3 pts)

Solution. We compute the limit of the sequence of terms $a_n = \frac{1}{2n^3-1}$ and get $\lim_{n\to\infty} \frac{1}{2n^3-1} = 0$, and therefore the divergence test is inconclusive. □

3. Apply the **ratio test** to check for convergence or divergence. What is the conclusion? (3 pts)

Solution. We compute

$$\rho = \lim_{n\to\infty} \left| \frac{\frac{1}{2(n+1)^3-1}}{\frac{1}{2n^3-1}} \right| = \lim_{n\to\infty} \frac{2n^3 - 1}{2(n^3 + 3n^2 + 3n + 1) - 1}$$
$$= \lim_{n\to\infty} \frac{2n^3 - 1}{2n^3 + 6n^2 + 6n + 1} = 1,$$

so the ratio test is inconclusive. □

4. Apply the **comparison test** to check for convergence or divergence. What is the conclusion? (*Hint: Your answer in problem 1 is important to find a candidate series to compare with.*) (3 pts)

Solution. The series almost looks like $\sum_{n=1}^{\infty} \frac{1}{2n^3} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n^3}$, which is a (multiple of a) p-series with $p = 3$, so it should converge. We therefore guess that the initial series converges, and want to compare it to the above one. Note

$$\sum_{n=1}^{\infty} \frac{1}{2n^3 - 1} \leq \sum_{n=1}^{\infty} \frac{1}{2n^3 - n^3} = \sum_{n=1}^{\infty} \frac{1}{n^3},$$

where the larger series converges because it is a p-series with $p = 3$. The comparison test therefore yields that $\sum_{n=1}^{\infty} \frac{1}{2n^3-1}$ converges. □