ON THE COMPLEMENT OF THE PROJECTIVE HULL IN C^n.

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ABSTRACT

We prove that if K is a compact subset of an affine variety $\Omega = \mathbf{P}^n - D$ (where D is a projective hypersurface) and if K is contained in a closed analytic subvariety $V \subset \Omega$, then the projective hull $\widehat{K}$ has the property that $\widehat{K} \cap \Omega \subset V$. If V is smooth and 1-dimensional, then $\widehat{K} \cap \Omega$ is also closed in Ω. The result has applications to graphs in $\mathbf{C}^2$ of functions in the disk algebra.

Let X be a compact set in $\mathbf{C}^n$. In [1], R. Harvey and the first author introduce a generalization of the polynomial hull of X, which they call the *projective hull of X in $\mathbf{C}^n$*, and denote by $\widehat{X}$. For $d = 1, 2, ...$ let $\mathcal{P}_d$ denote the space of all polynomials on $\mathbf{C}^n$ of degree $\leq d$. Then by definition a point z lies in $\widehat{X}$ if there exists a constant $C \geq 1$ such that for all d and for all $P \in \mathcal{P}_d$ with $\sup_X |P| \leq 1$, we have

$$|P(z)| \ \leq \ C^d.$$

Fix a point $a = (a_1, ..., a_n) \in \mathbf{C}^n - X$. For $d = 1, 2, ...$ set

$$\lambda_d(a) \ \equiv \ \sup\{|P(a)| : P \in \mathcal{P}_d \text{ with } \|P\|_X \leq 1\}$$

Note that a lies in the complement of $\widehat{X}$ if and only if for each $M \geq 1$ there exist arbitrarily large d with $\lambda_d(a) \geq M^d$.

We call a set X *non-algebraic* if every polynomial vanishing on X is identically zero. We restrict our attention to such non-algebraic sets. We study the function λ_d and, in Theorem 2 below, give a result on the complement of $\widehat{X}$ in certain cases, generalizing Theorem 9.2 in [1].