REMARKS ON THE ALEXANDER-WERMER THEOREM FOR CURVES

F. Reese Harvey and H. Blaine Lawson, Jr.*

Abstract

We give a new proof of the Alexander-Wermer Theorem that characterizes the oriented curves in $\mathbf{C}^n$ which bound positive holomorphic chains, in terms of the linking numbers of the curve with algebraic cycles in the complement. In fact we establish a slightly stronger version which applies to a wider class of boundary 1-cycles. Arguments here are based on the Hahn-Banach Theorem and some geometric measure theory. Several ingredients in the original proof have been eliminated.

Table of contents.

1. The Alexander-Wermer Theorem
2. A Dual Interpretation
3. The Remainder of the Proof.

*Partially supported by the N.S.F.

1. The Alexander-Wermer Theorem

We present a different proof of the Alexander-Wermer Theorem [AW], [W_2] for curves which uses the Hahn-Banach Theorem and techniques of geometric measure theory. Several ingredients of the original proof are eliminated, such as the reliance on the result in [HL_1] that if a curve satisfies the moment condition, then it bounds a holomorphic 1-chain. The arguments given here have been adapted to study the analogous problem in general projective manifolds (cf. [$HL_{3,4,5}$]).

Our arguments will also apply to a more general class of curves which we now introduce.