PROJECTIVE LINKING AND BOUNDARIES OF POSITIVE HOLOMORPHIC CHAINS IN PROJECTIVE MANIFOLDS, PART II

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Abstract

Part I introduced the notion of the *projective linking number* $\mathrm{Link}_{\mathbf{P}}(\Gamma, Z)$ of a compact oriented real submanifold Γ of dimension $2p-1$ in complex projective n-space $\mathbf{P}^n$ with an algebraic subvariety $Z \subset \mathbf{P}^n - \Gamma$ of codimension p. It is shown here that a basic conjecture concerning the projective hull of real curves in $\mathbf{P}^2$ implies the following result:

Γ is the boundary of a positive holomorphic p-chain of mass $\leq \Lambda$ in $\mathbf{P}^n$ if and only if the $\widetilde{\mathrm{Link}}_{\mathbf{P}}(\Gamma, Z) \geq -\Lambda$ for all algebraic subvarieties Z of codimension-p in $\mathbf{P}^n - \Gamma$.

where $\widetilde{\mathrm{Link}}_{\mathbf{P}}(\Gamma, Z) = \mathrm{Link}_{\mathbf{P}}(\Gamma, Z)/p!\deg(Z)$. An analogous result is implied in any projective manifold X.

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1. Introduction

In Part I we introduced a linking pairing for certain cycles in projective space as follows. Suppose that $\Gamma \subset \mathbf{P}^n$ is a compact oriented submanifold of dimension $2p-1$, and let $Z \subset \mathbf{P}^n - \Gamma$ be an algebraic subvariety of codimension p. The **projective linking number** of Γ with Z is defined to be

$$\mathrm{Link}_{\mathbf{P}}(\Gamma, Z) \equiv N \bullet Z - \deg(Z) \int_N \omega^p$$

where ω is the standard Kähler form on $\mathbf{P}^n$ and N is any integral 2p-chain with $\partial N = \Gamma$ in $\mathbf{P}^n$. This definition is independent of the choice of N. The associated **reduced linking number** is defined to be

$$\widetilde{\mathrm{Link}}_{\mathbf{P}}(\Gamma, Z) \equiv \frac{1}{p! \deg(Z)} \mathrm{Link}_{\mathbf{P}}(\Gamma, Z)$$

The basic result proved here is the following.