PROJECTIVE LINKING AND BOUNDARIES OF POSITIVE HOLOMORPHIC CHAINS IN PROJECTIVE MANIFOLDS, PART I

F. Reese Harvey and H. Blaine Lawson, Jr.*

Dedicated to Nigel Hitchin
in celebration of his 60th birthday

Abstract

We introduce the notion of the *projective linking number* $\mathrm{Link}_{\mathbf{P}}(\Gamma, Z)$ of a compact oriented real submanifold Γ of dimension $2p-1$ in complex projective n-space $\mathbf{P}^n$ with an algebraic subvariety $Z \subset \mathbf{P}^n - \Gamma$ of codimension p. This notion is related to *projective winding numbers* and quasi-plurisubharmonic functions, and it generalizes directly from $\mathbf{P}^n$ to any projective manifold. Part 1 of this paper establishes the following result for the case $p = 1$. Let Γ be an oriented, stable, real analytic curve in $\mathbf{P}^n$. Then

Γ is the boundary of a positive holomorphic 1-chain of mass $\leq \Lambda$ in $\mathbf{P}^n$ if and only if $\widetilde{\mathrm{Link}}_{\mathbf{P}}(\Gamma, Z) \geq -\Lambda$ for all algebraic hypersurfaces $Z \subset \mathbf{P}^n - \Gamma$.

where $\widetilde{\mathrm{Link}}_{\mathbf{P}}(\Gamma, Z) = \mathrm{Link}_{\mathbf{P}}(\Gamma, Z)/\deg(Z)$. An analogous theorem is implied in any projective manifold. Part 2 of this paper studies similar results for $p > 1$.

Table of contents.

1. Introduction.
2. Projective Hulls.
3. Projective Linking and Projective Winding Numbers.
4. Quasi-plurisubharmonic Functions.
5. Boundaries of Positive Holomorphic Chains
6. The Projective Alexander-Wermer Theorem for Curves.
7. Theorems for General Projective Manifolds.
8. Relative Holomorphic Cycles.

*Partially supported by the N.S.F.

1. Introduction

In 1998 Herb Alexander and John Wermer published the following result [AW_2].