MAT 550, REAL ANALYSIS
SUNY at Stony Brook, SPRING 99
Problem Set 3
Due on Friday, Febuary 12

1. For each $1 \le p \le \infty$ give an example of a function $f \in L^p(\mathbb{R}, dx)$ such that $f \notin L^q(\mathbb{R}, dx)$ for any $q \neq p$.

2. Give an example of a function $f \in L^1([0,1], dx)$ such that $f \notin L^2([a,b], dx)$ for any interval $0 \le a < b \le 1$.

3. Give an example of a function f on $[0,1]$ which is in every $L^p([0,1], dx)$ for every $p < \infty$, but which is not in $L^\infty([0,1], dx)]$.

4. Show that $1 \le p < q < \infty$ implies $L^p(\mu) \subset L^q(\mu)$ iff there is a $\epsilon > 0$ so that $\mu(E) < \epsilon \Rightarrow \mu(E) = 0$ for every μ-measurable E.

5. Show that $1 \le p < q < \infty$ implies $L^p(\mu) \supset L^q(\mu)$ iff there for every $\epsilon > 0$, X contains only finitely many disjoint sets of measure $\epsilon < \mu(E) < \infty$.

6. Show the following are equivalent:

   (a) $L^p(\mu) = L^q(\mu)$ for all $1 \le p < q < \infty$.

   (b) $L^p(\mu)$ is finite dimensional for all $1 \le p < \infty$.

   (c) $L^p(\mu)$ consists entirely of simple functions for every $1 \le p < \infty$.

   (d) X can be written as a finite union of atoms (i.e., X is the disjoint union $\cup_{k=1}^n X_k$ where each X_k has the property that for any $S \subset X_k$ either $\mu(S) = 0$ or $\mu(S) = \mu(X_k)$).