MAT 550, REAL ANALYSIS I
SUNY Stony Brook, Spring 1999
Comparing types of convergence

Six types of convergence on a measureable set E :

1. $f_n \to f$ a.e. iff there exists a set $A \subset E$ of full measure so that $f_n(x) \to f(x)$ for $x \in A$.
2. $f_n \to f$ unif. (uniformly)if given any $\epsilon > 0$ there is an n_0 so that $\|f - f_n\|_\infty < \epsilon$ if $n \geq n_0$.
3. $f_n \to f$ a.u. (almost uniformly) iff given $\delta > 0$ there exists a set $A \subset E$ such that $\mu(A) < \delta$ and $f_n \to f$ uniformly on $E \setminus A$.
4. $f_n \to f$ meas. (in measure) iff given any $\epsilon > 0$ there is a n_0 so that $\mu(\{x \in E : |f_n(x) - f(x)| > \epsilon\}) < \epsilon$ for $n \geq n_0$.
5. $f_n \to f$ in mean (or in L^1) iff f and each f_n is integrable and $\lim_n \int_E |f - f_n| = 0$.
6. $f_n \to f$ weakly iff $\lim \int f_n = \int f$.

If E has infinite measure, the following diagram shows which types of convergence imply which other types. Find a proof for each arrow and a counterexample for each missing arrow.

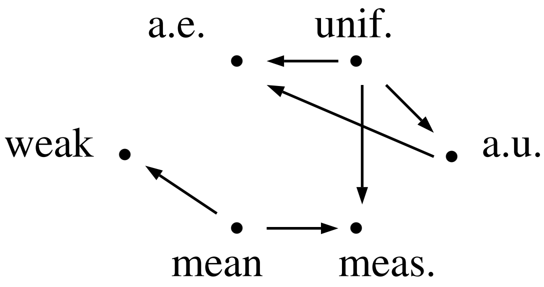

FIGURE 1. $\mu(E) = \infty$

If E has finite measure, the following diagram shows which types of convergence imply which other types. Find a proof for each arrow and a counterexample for each missing arrow.

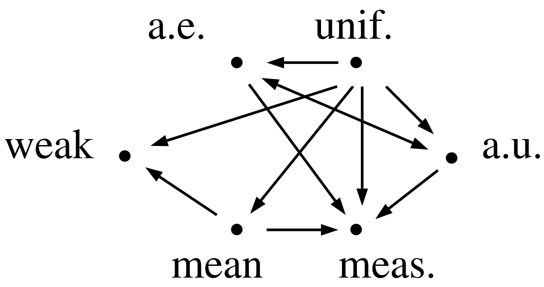

FIGURE 2. $\mu(E) < \infty$

If E has infinite measure and there is an integrable function g so that $|f_n| \le g$ for all x, the following diagram shows which types of convergence imply which other types. Find a proof for each arrow and a counterexample for each missing arrow.

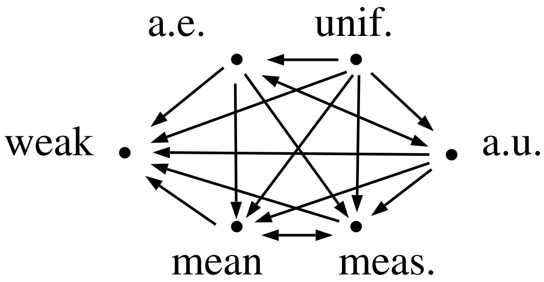

FIGURE 3. $\mu(E) = \infty$, $|f_n| \le g \in L^1$