PROBLEM SET 3

1. A function is called simple if it only takes on finite number of different values. If g is bounded and measurable, and $\epsilon > 0$ is given, show there is a measurable simple function f so that $\sup_x |g(x) - f(x)| \leq \epsilon$. Is this true if g is not bounded?

2. Suppose E is measurable set of real numbers and let $f(t) = m(E \cap (t-1, t+1))$. Show that f is continuous.

3. Suppose E is a closed set in the upper half-plane whose vertical projection onto the real line is surjective (onto). For each real number x let $y(x)$ be the closest point of $E \cap L_x$ to x (here L_x is the vertical line through x). Show that y is a measureable function, but need not be continuous.

4. Given a real number $x \in [0,1]$ let $x_n(x) \in \{0,1\}$ be its n-binary digit (if this is not unique, choose the epansion ending in all 0's). Let $f(x) = \limsup_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} x_n(x)$. Show that f is measureable. Where is f continuous? Can you guess what $\int_0^1 f(x)dx$ is?