PROBLEM SET 2a

1. Prove that the Lebesgue function F satisfies

$$|F(x) - F(y)| \leq |x - y|^{\alpha},$$

where $\alpha = \log 2/\log 3$. This is called a Hölder condition of order α.

2. If $E \subset [0,1]$ is a null set, and $f : [0,1] \to [0,1]$ is continuous, does $f(E)$ have to be a null set as well? Prove this or find a counterexample.

3. Let $X = \{x + y : x, y \in C\}$ be the set of sums of numbers in the Cantor middle third set. What is X?

4. Prove that if $\lambda > 0$ then $m^*(\lambda E) = \lambda m^*(E)$ where $\lambda E = \{\lambda x : x \in E\}$.

5. If X is set of finite Lebesgue measure show that $m(X \cap X + t) \to 0$ as $t \to \infty$. Here $X + t = \{x + t : x \in X\}$. Does there have to be a value of t so that $m(X \cap X + t) = 0$?